**Relaxation Time Approximation**

Basically going to copy and paste file from Stat Mech folder here. But thought it’s a good idea to present it in a different way. So going back to the Stat Mech folder and looking up the Classical NESM RTA (MF) file, we found that we could use the RTA equation and construct two self-consistent equations for the particle density and current, under a mean field approximation. The equations we found were,



where,



where d is the dimension. At T = 0 (or T << TF), we can approximate this for electrons as:



Note we can also write this as:



where ℓsc is the mean free path. Or if we’re dealing with classical particles – conduction electrons in semiconductors are an example – then we’d have:



We can reproduce a few nice results from these equations. Consider the time-independent case.



We see the current is driven by two mechanisms. The first is a density gradient, which produces a current via diffusion. And the second is a ‘potential’ gradient, i.e., force, which produces a current via force. If we presume there is no density gradient either, then we just get:



We can solve this by crossing and dotting both sides by **B**. We’ll cross first,



Using,



we have,



Filling this in,



And now dot both sides of our equation by **B**,



Now fill our dotted equation into our crossed equation,



and now fill this into our original equation,



And now solve for **j**,



And let’s use the cyclotron frequency,



to say,



And we finally arrive at:



Again, I wonder about the sign of the E×B term. But in any event, we see we have a current that flows in the direction of E, and one in the direction of B, and then one (the drift current) perpendicular to both. We’ll also note that if τsc → ∞, i.e., the limit of no scattering, then term proportional to E drops out. And we’d just have the current going along B and the drift current. The one going along B appears to blow up in this limit, but in reality, it just means that our formula breaks down. If we go back to the beginning we’ll see that the current would accelerate along B (see EM folder/Charge Dynamics for instance). We’ll recognize the magnetic field dependent conductivity, i.e., the proportionality between **j** and **E**.



which reduces to the usual result when B = 0. Let’s allow time-dependence now, but for simplicity, I’ll presume B = 0 (can use same procedure as above if want to include B). Then our RTA equation becomes,



And let’s solve this equation, presuming a sinusoidal **E**(t) = Re(**E**e-iωt). I’ll use Laplace Transform,



Now we’ll solve for .



And now take the inverse transform,



Note Re takes Real part of everything to the right. So now we have:



The first part is just the evanescant response of the current to the field. We’ll recognize the steady state term (second term) as the complex time-dependent conductivity.



which we’ve seen before. Finally, let’s go back and allow a density gradient, but just look for the steady-state oscillatory response to an oscillatory electric field (no B). So we go back to:



And let n(r,t) = Re(n(q,ω)eiq·r-iωt], ρ = en, **j**n(r,t) = Re[**j**(q,ω)eiq·r-iωt], **j** = e**j**n, **E**(r,t) = Re[**E**(q,ω)eiqr-iωt]. Filling these in, we have:



which is,



Filling the former into the latter, we have:



Let’s take the tensor **D** to be isotropic **D** = Dδij. Then,



And so finally,



We can write this equation as:



And so this makes the effective conductivity, when we include the possibility of charge density fluctuations, equal to:



Of course this reduces to our normal result, when D = 0.