**Electric Susceptibility**

So I guess we’re doing the electric susceptibility stuff. This time we’re allowing time-dependence. The formalism is pretty much identical to the static dielectric stuff, and is found in the EM folder too. We would need to use non-equilibrium thermodynamic methods to calculate these susceptibilities.

**Electric Susceptibility Definitions**

So the conceit is that we introduce a free charge/current density, which gives rise to free fields, and ultimately induces a responsive charge/current density. I’ll redo the analysis we did in EM file for metals, because of the slightly different pre-factors we’re using, mainly due to fact we’re in fake ‘Gaussian’ units now [basically can go from SI to fake Gaussian by ε0 → 1/4π, μ0 → 4π]. I’ll also allow time-dependent fields/susceptibilities, so we don’t have to redo the whole thing in the non-equilibrium file. Referring to the Units file, if we were to take the electric susceptibility equation:



and convert to fake Gaussian units, we’d get:



But instead, we’re going to write:



So this χirr is smaller than the χirr we defined in the EM folder by a factor of 4π, and the ‘advantage’ of doing so is it allows us to put things more easily in terms of the Coulomb potential’s Fourier transform. Anyway, this is equivalent to, taking the spatial/temporal Fourier transform of both sides:



Now how do all of these functions relate? We can use Maxwell’s equations, like was done before. We’ll use:



Taking the Fourier transform of both sides,



Plugging in our induced ρ,



And now we can define the proportionality between ρf and φ as:



where υ(q) = 4π/q2. This is called υ(q) because the Coulomb potential has the same Fourier transform, in Gaussian units, sans the factors of e. Must be careful not to confuse this υ(q) with the actual potential V(q) that *includes* the e’s. Now let’s relate the induced charges to the free potential. We have by definition:



and for the other, we have:



Now we’ll use this to define χ(q,ω):



There are lots of other ways we can write χ(q,ω)…



Finally, observe we can relate ρind to ρf via:



and we can put the total field in terms of the free field,



So,



**χ(q,ω) in terms of GF’s**

So we’ll recall the idea, covered in the EM folder and free electron file, that we have our electron system, and we put an external field through it, φext(q,ω). This induces a linear a response in the system which creates an induced charge density ρind(q,ω) and a consequent overall field/potential φ(q,ω). The proportionality between the induced charge and the field is called the ‘irreducible’ susceptibility. And we have:



We can define other factors then. Refering to those files again, we have:



and υ(q) = 4π/q2 happens to be the Fourier transform of the electric potential sans e’s, in Gaussian units. And we also have:



We’d like to evaluate χirr and its ilk. So the way we do it is to set up our interacting system, in our oscillating electric field. We’ll use the gauge where **A** = 0.



We’ll recall, δρ(q) = ρ(q) – ρ0, where ρ0 = N/V, N being number of electrons present. We have to use δρ, and not ρ, because our system is not just electrons, but electrons and jelly. And the external field will change the potential energy of our system only if there is a *net* charge density. As it is the electron density is canceled by the jelly density. I’ll go through the process of calculating ρind(q,ω) for the sake of seeing it again. We will end up with Kubo’s formula, as in the Stat Mech file. Going to let Ueq denote the time-development operator according to H = Heq = H012, and borrow the expression for f1 from the Stat Mech file. So



(where by VEM(Heq)(t) = Ueq†VEM(t)Ueq I mean the operator in the interaction picture). Continuing,



The exponential infinitesimal convergence factor in f1, designed to prevent quantum recurrences, limits the applicability of our results to the thermodynamic limit, because only here would energy spacings be infinitesimally small. Now we’ll plug in our expression for VEM(Heq)(t)…in position space, cause



Of course we can recognize a GF here – the (charge) density - (charge) density correlation function. We already encountered that in our study of collective oscillations. So we have that, taking account of our present definition of ρ (charge density) vs. the one we were using back then (regular number density):



where,



A Fourier transform gives:



and it is clear then,



Now from the collective oscillations file, we know the self-energy expansion of ΠR is:



and we know from the definition of χ(q,ω) – see EM file, or Free Electron file – that



and so it follows that:



Finally, since:



We have that:



which is how we were defining it anyway, before, in the Collective Oscillations file.