**Semiclassical Model**

Since diffusion is a transport property, we can expect it to evince signiatures of the metal-insulator transition, i.e., localization. We won’t be calculating D per seʹ, but rather making estimates on the diffusive behavior of the electron itself via path integrals. Note that these weak localization corrections apply equally well to light scattering in random dielectrics. This is why, for example, light tends to reflect more than ‘usual’ in a fog. Note our semiclassical model assumes we can treat the electron as a wave diffusing through the sample, and so limits applicability to λ << ℓ → 1/kFℓ << 1, which is the weak-disorder regime.

**Probability of Return**

We can use a path integral approach to demonstrate qualitatively how the quantum interference can prevent a particle from spreading over the entire sample. Consider the propagation of an electron from a point **r** to another point **r**´. The probability of propagation from **r** to **r**´ in time t is:



Note that the free particle propagator, below,



isn’t a pure phase, so we shouldn’t expect Ai to be a pure phase either. So we can express Ai as. Then we have:



where we presume paths have largely different phases, and wash out over the sum. Now consider the probability of propagating to the same point. This time, all paths’ phases will not randomize out; instead we will have that when path’ = pathTRS we get the same amplitude and phase.



So probability of return is double w/r to what we’d expect based on random diffusion. The constructive interference between these two paths is related to the **p** = -**p** singularity in the diagrammatic expansion. This means that more than random diffusion is behind the propagation process. And we could write the time development of such a particle as something like the following SDE:



where the a(r) term models tendency to collapse to the center (starting point). Turns out this will be enough to localize the particle in 1,2 dimensions, i.e., cause the particle to approach a stable probability distribution in the long time limit. What about 3 dimensions? A SDE could model classical localization (percolation), and I think that similar results have been determined, but quantum particles seem to localize more easily because of this P(r,r,t) phenomenon, even though they have the ability to tunnel.

**Add a magnetic field**

Let’s add a magneic field to the mix. The probability of propagation from **r** to **r**´ in time t is:



Then we have:



where we presume paths have largely different phases, and wash out over the sum. Now consider the probability of propagating to the same point. This time, all paths’ phases will not randomize out; instead we will have that when path’ = pathTRS we get the same amplitude and phase, sans a flux factor.



We can simplify the integral via:



and then we get:



So probability of return is still enhanced, but not by as much, and so B fields diminish the localization effect. So B supresses the return probability by effectively diminishing phase coherence. Note that if 2eΦBpath ~ π, then the enhancement will be zero. The sample length that would accommodate this equality is:



**Trying to add spin interactions**

What about adding magnetic impurities? The probability would be:



And the relevant action term is:



(I think) Where Θ is the time reversal operator. So,



I don’t think this is right. I think it’s supposed to be 1 – ( ). Or maybe it is correct. I have no idea. Well, anyway, when a spin rotates it introduces a – sign into S, which would suppress the return probability to point below the classical value, and so would anti-localize the particle.

Nonetheless, it apparently isn’t enough to compensate in Q1D as such metals still scale to an insulating state. I suppose that there are more corrections for such H’s – just like we had to go to 2nd (or 3rd?) order in PT to obtain the relevant S∙S effects on the conductivity.

**Rough Calculation of P(r,r,|t) in B = 0, S = 0, case.**

We will find that this tendency of the wavefunction to contract is sufficient to localize the particle if dimensional considerations are right. We may roughly estimate the localization length as the length at which the *enhancement* of probability of return is equal to 1 (or ½, whatever to this accuracy). The *enhancement* is simply the diffusive probability itself.

Let

λ = particle wavelength,

b = transverse dimension of the sample,

v = velocity of particle.

τ = m.f.t, after which we may consider the particle to be behaving diffusively

τL = phase breaking time, by which we may assume the particle has traversed the sample (or the phase coherence which produces this effect has dissipated). We presume that by the phase breaking time, the electron will have visited the transverse ends of the sample (if we’re in Q1D, Q2D), and so have equal probability of being anywhere along the transverse dimensions due to multiple reflections off of those ends, but that it will still be longitudinally diffusing.

Then,



Note we use the 3D diffusion term for small times, because this is before the electron has explored the boundaries of the conductor. But note T-guy says that we must consider b < ℓ for system to be QxD so that the motion is essentially ballistic in the transverse dimension. But this would mean that it is already essentially xD diffusing by the time of τ. And so we’d just ignore the first term and make τ the lower bound of the second term. We find the following result, using ℓ = √(Dτ), L = √(Dτ L).



which tells us,



So we find that it is inevitable for the electron to return to itself in Q1D, Q2D. I guess this is because the rate at which it’s sampling positions (linear roughly?), is greater than the rate at which its exploring *new* positions. The finite b extends our previous results to the quasi 1D, 2D dimensions. If we make the width b ~ kF-1 (meaning the width is a lattice spacing), and make the wavelength λ ~ λF, then asymptotic expression calculated earlier match these.

**Rough Calculation of P(r,r,|t) in B case**

Not sure how to modify the calculation to work out ξ in a B field. Problem is working out how to put that 1 + cos(2eΦB) into the integrand. Perhaps we could do something like,



Dunno. Plus the integral would suck. At least it should diminish with B, and eventually go to zero. We can imagine so because for large B there would be tons of paths that are large enough to encompass enough phase difference to kill the return enhancement. And so everything would average out again basically.

**Rough Calculation of P(r,r,|t) in BS case**

Not sure what we’d do here.

**General Comment**

So it is apparent that phase coherence is crucial to this phenomenon. And so it will be observed only when phase breaking mechanisms are suppressed so that their influence is much smaller than the sample. If we let τφ be the time for the electron’s phase coherence to be murdered – by phonons, electron-electron collisions, etc., then the sample length must be smaller than Lφ = √Dτφ.