**Excitations**

So going back to:



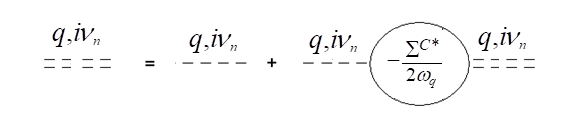
where,



It’s of interest to work out how a photon would propagate through a material. We kind of did this in the EM folder – both for metals (where we treated conducting electrons), and insulators (where we treated oscillating electrons, which is qualitatively similar to our oscillating lattice). There we examined wave solutions in those media and found that for a given energy/frequency, the momentum/wavevector of photon would change from medium to medium (this is basically Snell’s law). And the group velocity of the wave would change too of course, according to vg = ∂ωk/∂k. Additionally, there were a few things we could infer about the transmission/reflection and absorption of the waves, impinging from vacuum, say, onto those media. If k in the medium were purely real, then we’d get transmission and reflection. If it were purely imaginary, then we’d just get reflection. And if it were more generally complex, then we’d get both transmission/reflection, and partial absorption. The same should hold here. The only difference, as far as I can tell, between the non-quantized picture and the quantized one is that in the former, the material’s propagating EM wave modes are determined purely by the material itself, but in the quantized picture, one must simultaneously diagonalize the material + photon H to work out the propagating modes. That’s what we’ll do (or sketch out) now.

**Photon Self-Energy**

Now we want to examine the photon propagator. This will tell us the frequency at which a photon wavelength would propagate through the material. As usual we will look to formulate a self-energy.



Note the equation will be diagonal in the index.



which is:

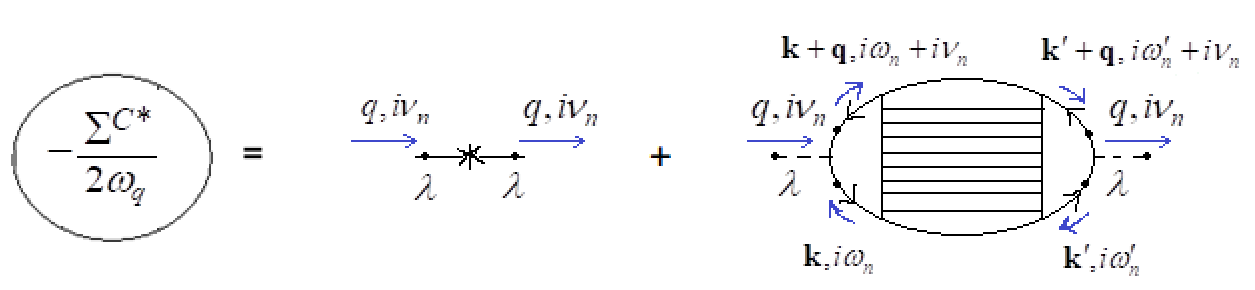


So now we have:



**Example**

Looks like we can conjure up an exact expression for the self-energy. Consider the general expansion will look like,



where the last guy is a polarization bubble containing all the (and none of the) e-e interaction terms. Note the bubble would not include any of the AqλA-qλ self energy terms within itself. This would be overcounting. Anyway, this would work out to (the product thing basically just means – all the stuff in between the two vertices):



Now **ε**(**q**,λ) is perpendicular to **q**, and so we can write this as:



But then we could also write this as:



The stuff to the right of the **:** is the current-current correlation, as might recall from any of the discussions of conductivity.



So then we can write:



The retarded self-energy would be:



Now we’ll recall from our discussion of the conductivity, that



So we can write the retarded self-energy as:



For a homogeneous isotropic system, we’d expect σ to be proportional to the unit tensor. Then we could say, like we did when discussing the conductivity,



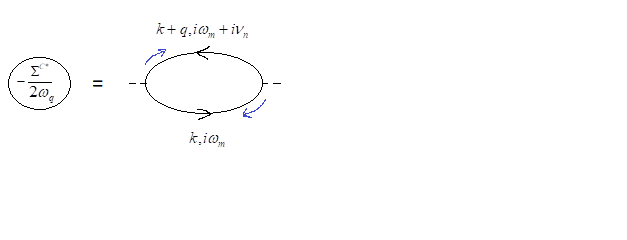
and write,



It makes sense that we can relate Σ to the conductivity, since as we’ll recall from EM (see Maxwell Equations Metal TD), Re(iσ/ω) determines the absorption spectrum.

**Example**

Let’s do the self-energy to first order (shouldn’t have the star guy inside current-current vertex since that would be including self-energy correction in the self-energy correction).



This is:



and so,



Since **ε**(**q**,λ) is perpendicular to **q**, we could write this as:



We’ll notice that the stuff to the right of the **:** is basically the first order approximation to the current-current correlation function [well with unperturbed GF’s – we have no disorder here] This makes sense because ImΣ ~ ImΠ ~ Reσ, and we know that the decay rate of a photon should be related to how quickly it’s absorbed, which would be related to Reσ. Anyway, we could continue,



For simplicity, one could calculate the polarization-averaged self-energy,



This doesn’t seem blindingly simple, but there we go. We basically calculated it in the Drude conduction file anyway (Metals/Impurities/Non-equilibrium/Conduction).