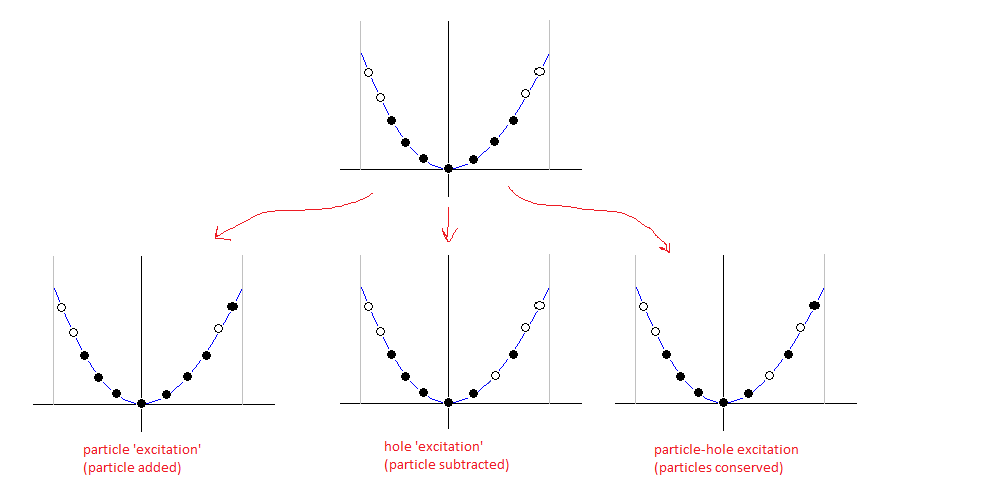
**Crystal Excitation Properties**

So when get to superconductors, we’ll find that a different perspective on the free Fermion excitations is useful. This is the Particle-Hole Model. So let’s take some time to consider it now in the free particle case.

**Excitations in Particle-Hole Model(s)**

Might note that given an N-particle system, adding and subtracting particles consitutes another eigenstate. Adding a particle we might call a particle excitation. Adding a hole (subtracting a particle) we might call a hole excitation, though really, we’d be decreasing the energy. And bumping a particle up from within the Fermi sea to outside of it (which would conserve particle number) is what we’d term a particle-hole excitation as it’s kind of a combination of the two. And this is the kind of excitation we’d normally have in mind when discussing system excitations.



Some more stuff. The 2nd quantized Hcrystal is usually written like this:



where εk would be the renormalized energy spectrum k2/2m\* or something. One could think of this as being written in terms of excitations above the zero-particle GS, |0>, i.e., the vacuum. Sometimes it’s more convenient to write it in terms of excitations above the N-particle GS, |F>, i.e., the Fermi GS. To wit, we could make the following manipulations:



where EGS = (3NF/5)EF basically. Now note that for k < kF, ck creates a hole, and ck† annihilates a hole (by destroying and creating a particle respectively). Let’s call a hole creator dk† and hole annihilator dk. Then we can write:



And we’ll note that creating a hole does eliminate energy so the (-εk) guy is appropriate. Also note that c’s and d’s would all anti-commute with each other since their k’s are different. We could write this more concisely as:

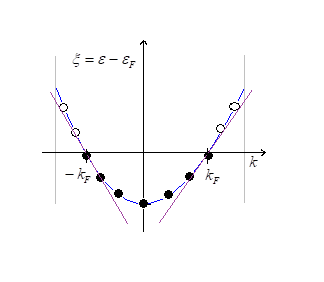


***Shifting zero-point energy to εF***

Often times, one changes the arbitrary 0-point energy scale (we already did anyway by ignoring ε0 above), and measures particle energies from the Fermi surface (εF = kF2/2m). So we’ll be implicitly considering the Hamiltonian,



I’m going to call it K, to distinguish it from H above, and also to make contact with the K that we use when calculating thermal averaged GF’s in the grand canonical ensemble. Single particle spectrum is illustrated below:

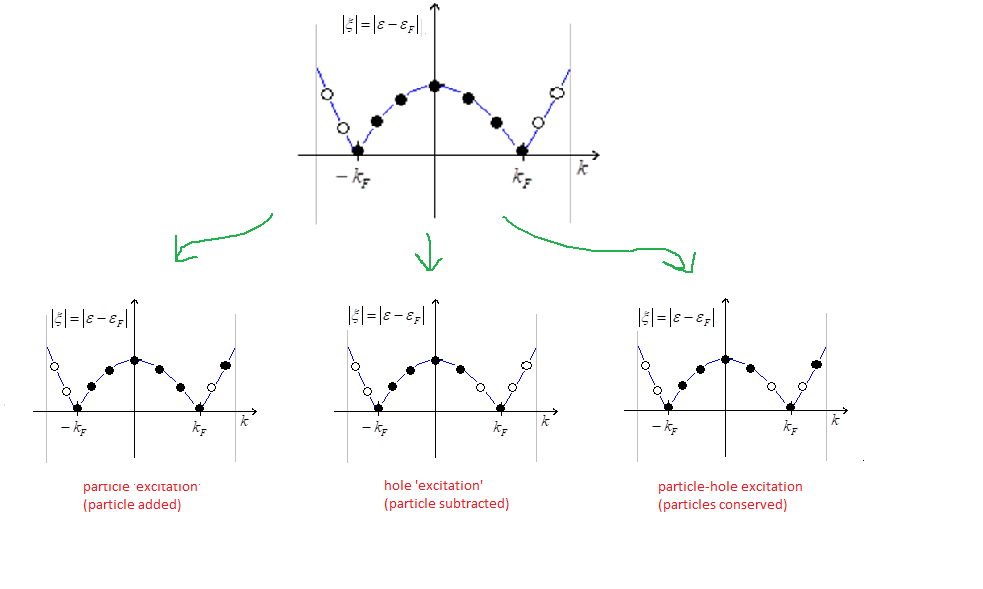


Not far from the surface one could linearize the spectrum, and write:



Linear spectra are easier to work with, and in 1D, enable exact solution of the electron-electron interaction via ‘bosonization’.

So measuring energies from εF is also useful in the way that it clearly delineates hole excitations from particle excitations. First, let’s note that the Fermion ground state |F> consisting of particles filled up to εF would have the most negative energy possible, for any state in the system, as adding another particle (k > kF *necessarily*) will now add a positive energy k2/2m – εF, whereas subtracting a particle (k < kF *necessarily*) will subtract a negative energy k2/2m – εF, which is equivalent to adding a positive energy, and so from this perspective, adding either particles or holes increases the energy either way, by |ε – εF|. Can think of it in the sense that in constructing K, we’ve implicitly set the chemical potential to μ = εF, and so ‘equilibrium’ is attained by filling the gas up to that level, and any deviations from that level will upset equilibrium. And in this context that means higher energies. Well, the spectrum would look like this below:

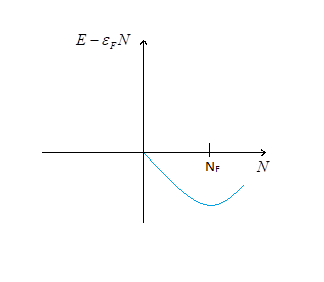


(the fact that the end of the band is at energy |ξ| = εF is coincidental) Note the filled in circles represent the ground state. And in this case at least, adding up the energies of the filled in circles would give us the ground state energy w/r to the vacuum. But that’s not the essential feature of the dispersion relation. Rather it shows us how to calculate the energies of excitations above the ground state. Excitations above the ground state will comprise either a particle added outside the Fermi surface (1st diagram), or a hole added within the Fermi surface (2nd diagram), or both, i.e., a particle-hold excitation (3rd diagram). And to work out the energy of that excitation we add up the energy value of each *particle outside* the Fermi surface and each *hole* *within* it.

Seems kind of counter-intuitive that adding holes will increase energy (but see comment about chemical potential interpretation). But it does, and is due to measuring each particle’s energy from εF. For instance, the shifted energy as a function of particle number is given by (in 3D, and using results below):



Plotted below (well it’s what I imagine it looks like):



It has a local minimum where N = NF [and energy is E = (3/5)NFEF – NFEF = -2NFEF/5] which is the number of particles that makes the Fermi surface have energy μ = εF. So can see that adding particles so that N > NF does increase energy, but so does subtracting particles so that N < NF. So the counter-intuitive notion of positive hole energies is just an artifact of measuring energy from εF. We can make this more explicit by, like we did above, writing H in terms of excitations above the GS, instead of in terms of excitations above the vacuum. Then starting with this:



where εk would be the renormalized energy spectrum k2/2m\* or something, we’d write:



Again, for k < kF, ck creates a hole, and ck† annihilates a hole (by destroying and creating a particle respectively). So we call a hole creator dk† and hole annihilator dk. Then we can write:



And we’ll note that creating a hole does increase energy, as argued above, by εF -εk. We could write this more concisely as:



Another nice thing about measuring energies from εF is that this automatically makes the lowest energy state the one with NF particles, whereas the state that had the lowest energy when we wrote H as we did above with εF = 0, is was the state with 0 particles.

**Density of states, etc., in the particle-hole model(s)**

Let’s reprise some of those calculations in the particle-hole model. Consider first:



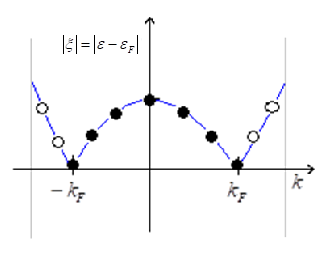
And let’s work out the density of states/excitations. Let εF = kF2/2m.



Note ε ranges from (-0,-εF) vis a vis the hole spectrum, and (εF, ∞) vis a vis the particle spectrum. Now let’s look at the other guy.



whose energy spectrum |εk – εF| looks like this:



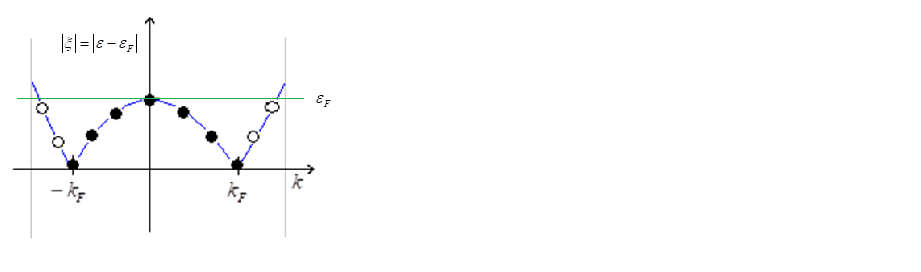
which are excitations above the GS. Then what is the density of states of these excitations? I’ll stick with 3D. And below, I guess I should’ve written ξ as |ξ|, but didn’t want to bother, so…I’ll just treat ξ itself as a positive variable.



Since ξ is positive, as is εF, we can write this as:



This seems plausible, as there ought to be two contributions to the density of states up to the green line (which would be a height εF above the axis), and just one contribution afterwards.



What’s the d.o.s. at the Fermi surface, ξ = 0? We’d have:



(only one contribution - ‘cause of that inequality restriction – otherwise overcounting) And this is what it’s supposed to be. Well, really, we ought to include the other contribution since density of states should be good over a range, and any range would include the other branch. So,

