**Heat Capacity**

**Heat Capacity via BCS model**

So in last file we saw that our Landau potential was:



And to get the heat capacity, we could do a couple things at this point. We could differentiate to get S = -∂L/∂T, and then differentiate again to get Cv = T∂S/∂T. Or we could form the grand partition function:



and calculate the internal energy,



and differentiate w/r to T to get the heat capacity. But instead, recall that in the free fermions stat mech file, we derived from L an expression for the entropy. We can do the same here, merely switching letters, to obtain,



The heat capacity follows, then:



So,



Filling in the distribution,



Now gotta work out the derivative. So note (allowing x to depend on β):



Therefore,



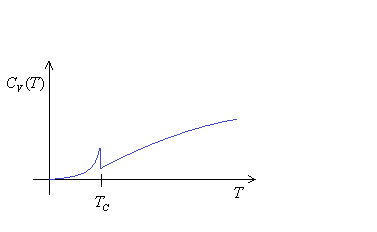
and then apropos the ξ derivative,



So,



The first term in the brackets is the normal non-interacting contribution, and the second term is the super-conducting condensate contribution. If we plot this, we get something like:



For T << Tc we have exponentially damped CV ~ (∂/∂T)(2Δ)e-βΔ ~ (2β2Δ)e-β(2Δ). This is due to the 2Δ energy gap which presents an activation barrier of sorts. Any finite energy level system will display similar behavior. After Tc, the energy barrier is gone as Δ → 0. And the system reverts to normal Fermi liquid behavior. The border between these two regions has a discontinuity. Note we also observe a discontinuity in the specific heats of magnetic systems that undergo phase transitions – see stat mech file. The critical exponent associated with this transition is cV ~ 1/|T-Tc|α where α = 0, which is the typical mean field result. And unlike usual, seems to accurately describe the actual situation. We can work out the discontinuity. This is:



where I ignored the k ‘because I think I’ve ignored it in the Δ below, and I should here, to be consistent. So now we’ll borrow the Δ result from a previous file,



We have, evaluating T at Tc after taking the derivative,



So that brings us to, noting that at Tc, Δ → 0 and k → ξk:



Now this sum over states is basically Vρ(εF)kBTc, because the n product is non-zero only within an interval of about kBTc about the Fermi surface. Oh yeah, ignoring k (kB) still. So now we have:



And we can divide by V to get the *specific* heat discontinuity:

