**Excitations**

Going to redo our analysis, using Mahan’s potential model. But it looks like it’s not going to work out too well. Here goes…

**Mean Field Theory**

We’ll use the BCS model, as in Mahan.



The focus of our efforts will be to attempt to determine the single particle Green’s function.



And we will use MFT to do it. It is interesting he says that mean field theory can describe SC so well. He says that it usually does a good job when the range of the interaction is large. He says that here the interaction is not so large but the radius of the ‘bound states’ of the pairs *is* so it works rather well. Our options are to apply MFT directly to H, or to the GF PDE instead. The easiest option is the latter. They say. We will need to examine three different GF’s to get self-consistent equations. These are:



These are the only unique GF’s (see the Symmetry Considerations File). Well, actually G↑↑ = G↓↓, but I’m going to leave as Gσσ I guess. To get the equations of motion we need to figure out the equation of motion of the operators. So putting in a chemical potential, and going to imaginary time, we can update the equation of motion formula in the QM/Many Particles/2nd quantization file. We have:



and also that (should look at real time version, and then complex conjugate everything, rather than try to complex conjugate the previous line)



And so the equation of motion of our GF’s will be:



the two δ’s combine to give the anti-commutator of cc† which gives 1 (can set τ´ = τ in first one, cause of δ, and then anti-commutator is preserved by time-development). Then,



and finally,



Now we can do a mean field approximation by using Wick’s theorem on the fully time-developed operators. Of course this isn’t strictly valid. We’ll include contractions of cc and c†c† as the states against which we’re taking expectations are superpositions of many-body states.



Now in a translationally invariant state, we should have momentum conservation in these expectations (see Symmetry Considerations file). Should have spin conservation too (also see Symmetry Considerations file). So we can say,



(note even though F, F† are odd in sgn of σ (see Symmetry Considerations file), since there is product of them, it doesn’t matter, and so we get the same result whether σ is up or down) Now the blue guy is the Hartree diagram, and the red guy is the Fock diagram. These guys will just renormalize the chemical potential, so we’ll absorb them into it: ξk → ξ´k = εk - μ´ I guess. And these n’s don’t really depend on τ as this is <Tre-βKeKτc†e-KτeKτce—Kτ> = <Tre-βKeKτc†ce—Kτ> = <Tre-Kτe-βKeKτc†c > = <Tre-βKe-KτeKτc†c >

= <Tre-βKc†c >. Also don’t expect F to depend on sign of k arguments (consequence of Parity symmetry), so,



Now define,



and we have:



Okay. Now let’s get the equation for F (σ is up)



Then we apply Wick’s theorem again, as part of the mean field approximation:



and,



The blue and red guys are the Hartree and Fock terms again. We will again absorb them into ξk → ξ´k. And we’ll also recognize Δ. So we’ll have:



And now the other guy too (σ pointing up again).



Then the δ guys cancel due to anti-commutator.



Then applying Wick,



and,



We absorb those terms into ξk → ξ´k (think it’s legal to switch q → -q). And then have:



Now recognize,



(can switch q → -q pretty sure, ‘cause V(q) would be symmetric) and so we have:



Altogether, our three equations are, setting τ´ = 0,



These 3 equations are known as Gorkov’s equations. Now let’s take the Fourier transform, ∫0βdτ eiω\_n·τ, of our three equations. See the next file for more on taking the transform. We get:



The first and last can be solved.



We find:



and F can be determined from these. So altogether:



Now Δ is determined by the self-consistency requirement (the ‘gap equation’). Recall we defined:



In terms of the temporal Fourier transformed FC\* highlighted above, this is:



We can do the frequency sum using the Matsubara thing,



So,



Since:



We can say,



Let’s go back to,



Now filling in the potential,



It doesn’t look like our former ansatz, that Δ(k) = Δθ(|ξ´k|<ωD) is tenable in this model. Rather we’d have to presume Δ is just constant for all wavevectors, Δ(k) = Δ (this is what Mahan does). Well then that wouldn’t seem to work either. Uh. Hmmmm. But Mahan pretends it does.