**Non-Equilibrium Properties**

So now we’ll run through a calculation of the absorbtivity and DC conductivity, with impurities included in the calculation. So H is given by,



where,



and where A is the vector potential, and jp the paramagnetic current density,



And recall in the Metal/Impurities/Conductivity folder we derived the following relationship,



and,



and from GF excitations file,



and also from the GF excitation file, I’m going to use the simple Born approximation to the disorder averaged GF,



where,



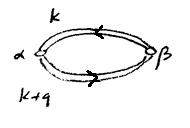
and where we take Δ to be real, and recall the Pauli spin matrices,



and **1** is the unit tensor, which I’ll leave off. Assuming Δ is real still.

**General formula for absorbtivity Reσ(q,ω)**

So let’s work Reσ(q,ω) out. We’re gonna repeat our calculation from before, w impurities this time. Now we should expect no absorbtivity for ω < 2Δ, but should expect some afterward. So going to do the diagram below. Basically the first order diagram for the current-current GF. But we’ll use the full Born approximation disorder averaged GF’s. Everything will be the same as last time, except replacing the bare GF’s (i.e. fully interacting, but no A’s, etc.) with the disorder averaged GF’s. We’ll use the self-consistent Born approximation for the disorder averaged GF’s.



So we have:



where



To evaluate S, we have to do that Matsubara sum over the frequencies. So recall the general technique elaborated on in the Stat Mech Math Appendix,



And so we have:



But given form of , we can see that it’s pretty gnarly to find the residues of this guy and execute the sum. So instead of filling in our G, we’re going to use its spectral representation instead (like we did before), and then do the sum over residues that way. So the complex frequency (i.e., analytically continued) GF is:



and **A**k(x) is the spectral function. And so we can say,



recalling nF(x) is periodic in boson frequencies. Now let’s analytically continue, and go back to Re**σ**(q,ω):



And now use 1/(x+i0+) = P(1/x) – πiδ(x). So,



Presuming we have an isotropic medium, we have σαβ = σδαβ, where σ would necessarily be (1/3)Tr[σαβ]. So let’s assume so, and then,



So,



**Absorbtivity Reσ(q=0,ω>0) @ T=0**

Now let’s focus on the absorbtivity. We’d like to see that it’s zero for ω less than 2Δ, when T = 0. So as T → 0, the nF(x) – nF(x+ω) term becomes just 1 within window x ∈ (-ω, 0). So can say,



Now we’ll recall A(ω) = -(1/π)GR(ω), so,



and clearing denominator,



Now let’s take the Trace thing,



Maybe go to q = 0 now.



Giving up for now. But we’re supposed to get something like,

Diagram

Description automatically generated

There is a δ function at ω = 0, just like we got in the clean (no impurities) case. And there is supposed to be a gap still between 0 < ω < 2Δ (I don’t see how the gap will come about yet). But otherwise, we should get absorption for ω > 2Δ.