**Weak Disorder**

Let’s do a few more examples, wanna get some GF’s in real position space.

**1D Real Space Green’s function to first order with δ potential**

Consider once again scattering of noninteracting electrons by the zero-range impurity potential. Ignoring the real part of the self-energy, the disorder-averaged Matsubara Green’s function is approximately.



Let’s find the real space Matsubara Green’s function. I’ll approximate τk as a constant τF.



Where k+ is the pole in the u.h.p.

*Recalculation with Fermi surface linearization approximation*

Next we’ll try the familiar trick of linearizing the spectrum and comparing that ‘easier’ calculation to our exact one here. So again the spectrum is roughly,



and so,



And now we’ll extend the integral to negative infinity since the main contribution will come from k ≈ kF (which ought to be fairly far from k = 0).



Now, to save us some work, we’ll note that this function is clearly a function only of the absolute value of x, and we’ll define,



So we can write,



Comparing with the exact result, we’d find that it is accurate generally speaking only for energies close to the Fermi level, and inverse lifetimes also small compared to the Fermi level. Next, we would like to calculate the (complex) time dependence of the Green’s function. So, constructing the inverse Fourier transform of out result,



Now we can see one advantage of the linearization scheme. Since it keeps the energies only to first order, it allows explicit summation as a geometric series. So working out Ωn, ωn explicitly in terms of n…



and,



So then,



and, recalling ω0 = 2π/β,



Collecting all the imaginary parts. Now let’s calculate the number density for this one dimensional gas. The number density is simply,



So,



Let’s consider the small T limit,



So we have density oscillations? Interesting.

**1D Real Space (on half line) Green’s function to first order with δ potential**

Consider a non-interacting one-dimensional Fermi gas confined by an impenetrable barrier to the half-line x > 0 (with impurities). We would have some clues as to what to use for basis functions from solving the single particle Hamiltonian in the presence of the potential barrier. Normalized eigenfunctions are indeed,



Note that k is actually restricted to be positive because negative k is just the same eigenfunction (to w/i an unimportant multiplicative constant). So we would first off try to form the position annihilation operator as,



But we want to make contact with the free particle creation operator. So we just say,



The normalization is kind of screwy, and ψ(x) = -ψ(-x). So do we care about what is happening for x, k < 0? Anyway, we’ll now calculate the single particle Matsubara Green’s function on the half line.



So,



Next we note that since G is a function only of |x-x´|. So,



Now we can calculate the number density using our previous result for the 1D Matsubara Green’s function. We need to calculate,



This is (taking small x limit in the first one)



Expanding for small x, we’d have,



So we see that the density is zero at the boundary, as we’d expect certainly. This effect should be due to the action of the potential barrier, and not from the impurities themselves. Expanding for large x, we’d have,



And here we see ‘Friedel’ oscillations of the number density. Finally we take the limit T → 0.



Hmmmm……