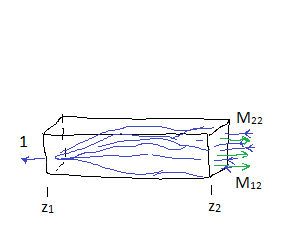
**Tartakovsky Summary**

Tartokovsky developed a more microscopic approach. He relates the transfer matrix, **M**, to wavefunction ψ within the medium, and then, via the Schrodinger equation, to the random potential U(**r**) driving its evolution. To start, he considers a sample situated between position z1 and z2, and examines how **M** will change as we vary either one of the two coordinates.



Note he defines **M** somewhat differently than convention. His **M** (LHS of equation) relates to the conventional **M** (RHS of equation) via:



For the latter case we consider the development of a wavefunction, ψ(n)(**r**) consisting of unit current passing leftward through the nth channel. Solving the Schrodinger equation, and projecting onto the (free) transverse eigenbasis, we have:



The second order equation can be reduced to two first order equations, which will decouple the boundary conditions, by design, utilizing the following definitions:



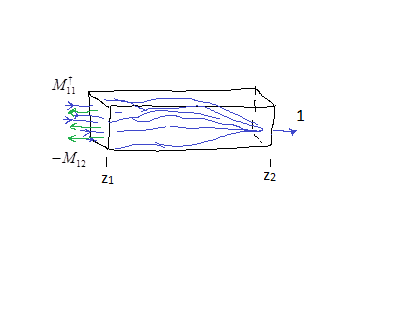
Upon which, we obtain the following equations, written in matrix notation (k = diagonal matrix km):



These equations now have the obvious interpretation of embodying the development of the two independent blocks M22, and M12 of the transfer matrix. From these, an evolution equation for the transfer matrix itself can be written down:



As alluded to earlier, this process can be repeated, but this time considering z1 to be moving, and z2 fixed. We start by writing down the solution to the Schrodinger equation for the following setup, the case of perfect transmission into the nth channel on thee right.



The result is:



With the stochastic differential equations written down, the next step is to specify the statistics of the potential. This is taken to be white noise:



where:



Note that, sans the author’s choice of phase factors embedded within **M**, which are responsible for the O(dz) term, this SDE is the same as the white noise model in the Appendix. We’ll also observe that, like Mello’s model, we find the fluctuational variance of **M** is proportional to **M2**. This makes sense in that we expect the fluctuations of **M** to grow as λ, and hence **M**, gets ‘larger’. All in all, we see the transfer matrix can be written as an Ornstein-Uhlenbeck process. Moreover, it is appropriate to interpret this as a Stranovich process since the random quantities have been so far been presupposed to follow the normal rules of Calculus. At this point, following Appendix B, one could write down the Fokker-Plank equation for the evolution of **M**’s probability distribution function. But again, this is of questionable utility. Rather, like Mello, he uses it to examine the evolution of the transmission/reflection matrices. These can be obtained by relating them to the blocks M11, M12. The equations for the transmission and reflection coefficients are:



and,



With these equations, the stochastic evolution of any function F(t,rʹ) may be computed via straightforward differentiation. Before taking the *average*, however, there are two caveats. First, one has to be careful to recast dF/dz into Ito form (see Appendix B) so as to circumvent the correlations between the white noise term and its multiplicand. Secondly, one will want to identify and eliminate the rapidly fluctuating ‘fast’ terms in dF/dz that average to zero. These can be identified by going to the ‘interaction picture’ of the above evolution equations:



Substitution of these expressions into dF/dz will reveal the terms which average to zero [this is analogous to the step that gives rise to the Cab,cd term in the Appendix model]. Once this is done, one obtains results similar to Mello’s for <t>, and <rʹ>. As before, it is of more interest to obtain information about the transmission/reflection probabilities <|tab|2>, <|rʹab|2> , but also as before, the equations for these quantities, inevitably involve higher moments, and so cannot be closed. A simplification of the equations is afforded if one goes to the Q1D limit and assumes that all transmission probabilities are of order 1/N, albeit with an intensity that is certainly sample length dependent. Making this assumption, along with occasional use of the mean field approximation, he is able to self-consistently solve for these quantities. In the asymptotic limit z → ∞, he finds:



where ρL(μ) is a function defined in the text. He uses this decidedly non-isotropic result to obtain Ohm’s law, including the skin depth correction. To obtain the general statistics of the conductance, he looks to examine the evolution of the probability distribution of the N independent generalized traces τk = Tr(Tk), where T = tt†. Unlike what happened for ρk = Tr(λk) in Mello’s model, the stochastic evolution equations for these traces do not readily reduce to functions merely of the same set of traces. So to circumvent this problem he considers the evolution of the *average* of *all* moments of these traces: d<τ1s1τ2s2…τNsN>/dz, where sj can range from 0 to ∞. Under the same prior assumptions, that transmission amplitudes are ~ 1/N, this *does* work out to particular linear combinations of elements of this same set of moments. Using these equations he is able to reproduce many standard results. In the metallic regime he determines the transmission eigenvalue density ρ(T), weak localization corrections to the conductivity and universal conductance fluctuations, as well as the Gaussian nature of the probability distribution Pz(g). In the insulating regime, assuming eigenvalues are well separated, he is able to write down an approximate evolution equation for Pz(g) and show the solution is indeed log-normal. Finally, as discussed in the Appendix, if one knows the evolution of the average of all moments of a set of quantities, then one may write down a Fokker-Plank equation for their probability distribution. In this manner he writes down an evolution equation for Pz(τ1,τ2,…,τN) (like was the case for Mello’s analogous equation, this will invoke traces τk>N, which can in principle rewritten in terms of some combination of τk<N).



[ξ = 2Nℓ/3], and shows that it is equivalent to the DMPK equation, putting the latter on a firmly microscopic basis. It would be worthwhile to see if the method could reproduce the full localization behavior of a conductor.