**Excitations**

So I want to explore the consequences of symmetry apropos the GF’s….

**BCS Model**

We’ll start with the BCS model:



where,



and we’re implicitly restricting the k and k´ vectors to the neighborhood of the Fermi surface. And we’re considering the following GF’s.



I’m going to look at the consequences of various symmetries, and see what they say. To that end, let’s consider more generally,



**Consequence of Translational Symmetry**

From the QM/Identical Particles/2nd quantization in position space file, we found these properties of momentum creation/annihilation operators.



Let’s verify our H has translational symmetry,



So it does. What consequence does this have for our GF? I guess it means that they must ‘conserve’ momentum. So in other words, consider we had two different momenta (and spins):



And the only way this equation could be true is if p = q. Can see that the same considerations would require that F and F† must be defined with q = -p. And so they are.

**Consequence of Spin Rotation Symmetry**

Now let’s consider a 180o rotation of spins,



Checking if H is invariant w/r to this operation:



So it depends on what Δσσ´ will do when flip its indices. If we do have this symmetry, then we can say,



and similarly for the complex conjugate. In that case, we could say,



which is our original Hamiltonian. So it is at least self-consistent to say that we have this symmetry. But it’s also self-consistent to say we don’t. And as for the GF, this implies,



So we have: G↑↑ = G↓↓, and G↑↓ = -G↓↑. And for F we also find: F↑↑ = F↓↓, and F↑↓ = -F↓↑. Same for F†. Now consider π rotations about the x-axis. We found in that position space 2nd quantization file,



and so H would go to:



In order for this to work out, we’d need, Δ to be odd in its spin indices. Is it? Well supposing D commutes with H, so that we do have this symmetry:



in which case we can say,



So then this requirement is consistent with H possessing this symmetry. But this would be in contradiction with the previous requirement on Δ imposed when we demanded it make the Hamiltonian possess symmetry upon π rotation about y-axis. Unless we suppose that Δσ=σ´ = 0. In any event, if we presume such a symmetry, then we’d have:



So we have: G↑↑ = G↓↓, and G↑↓ = G↓↑. Doing this with F, we’d find: F↑↑ = -F↓↓, and F↑↓ = -F↓↑.

**Consequence of Parity Symmetry**

What about Parity?



Let’s see if H commutes with the Parity operator.



So parity doesn’t seem automatically conserved. But if we suppose it is, then we can say:



in which case, our H would come to:



And consequence of this on GF? Well, as we expect,



the GF’s are even in their p argument. The same considerations will lead us to conclude that F and F† are also even in their p arguments.

**Consequence of Time Reversal Symmetry**

Finally we’ll consider time-reversal symmetry. Again, from that QM Identical Particles/2nd quantization in momentum space file, we had:



And let’s verify that H has time-reversal symmetry



So it doesn’t look like TRS is automatically a given either. It all depends on the properties of Δσσ´(k). But say we did have it. Then must be that:



And the consequence for the GF would be:



So we have: G↑↑ = G↓↓, and G↑↓ = -G↓↑. And for F we also find: F↑↑ = F↓↓, and F↑↓ = -F↓↑. Same for F†. These conditions are the same as for π rotation about the y-axis. In any event, I don’t think we assume this to be true.

**At least one Property of Δ**

Going back to:



We do have:



**Some relationships between the F’s, and F†’s**

Another thing we can get away with saying, is,



Similarly,



Looks like I used the fact that H is bilinear and so we can just commute/anticommute them at will (w/ requisite -ε cost). But this follows more generally from Parity symmetry, and the property in the Stat Mech/GF Formal Properties file, that:



So either way,



Also, from the Statistical Mechanics folder/GF Formal Properties file, we know that:



which implies,



**Some relationships between the G’s**

Wanna check out the G’s too. From the Stat Mech folder/GF Formal Properties, we have:



which implies (upon taking complex conjugate of both sides):



So,



So there are evidently six true d.o.f. here, 3 each for G and F.