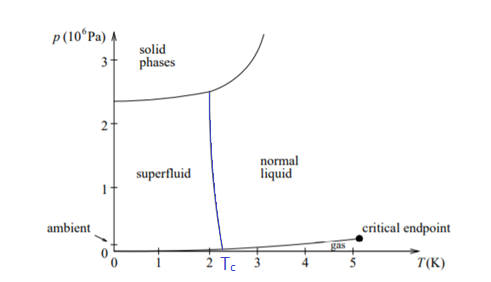
**Thermal Equilibrium Properties**

Here’s a rough phase diagram.



**Condensate Fraction at low T**

For T < TC, there will be a fraction, ns, of particles in the condensate (the superfluid part) and a fraction out of it (the normal part), nn, just as for the non-interacting fluid. Of course n0 = nN + nS. At Tc, ns = 0, and nN = n0. As T goes from Tc to 0, ns → n0, and nN → 0. This (gradual) collapse into a bound state is similar to how electrons in a gas will condense into a massive (singlet) bound state in a superconductor. And also how non-interacting electrons will gradually condense into the Bose-Einstein condensate as we lower T from Tc → 0. The condensate will exhibit super-flow which is flow without any resistance/viscosity. This is analogous to how the electrons in a metal will exhibit a super-current in their bound state. We can work out what ns(T) and nN(T) are by calculating the thermal averaged velocity expectation. Since it is only the condensate that exhibits superflow, this should take the form,



So let’s revisit the momenta of those low energy excitations in the previous folder (Feynman). They were:



and nk is just how many k-excitations we have. Recalling the energy of an individual k-excitation is εk,v = ω0(k) + **k**·**v**, the thermal averaged momentum of our fluid would be:



where nB(z) is the usual bose distribution function, with μ = 0. We’ll recall that near T = 0, μ = 0, as argued in the Stat Mech Independent particles, and Bose gas files. For instance, the number of particles in the Bose-Einstein condensate is given by:



So z → 1 as N0 → really big, and so μ → 0. Anyway, now to make further progress, we assume **v** is small and expand the distribution function:



Filling this in, we have:



where, since ω0(**k**) is isotropic, the first integral vanishes. And n0 = N/V of course. Now we can simplify our P expression,



In the last line, we’re saying that ∫d3k kx2∂n/∂ω = ∫d3k ky2 ∂n/∂ω = ∫d3k kz2 ∂n/∂ω since there is no prefered direction, i.e., ∂n/∂ω is isotropic. And so we can replace ∫d3k ki2 ∂n/∂ω with ∫d3k (kx2 + ky2 + kz2)/3·∂n/∂ω. And then we recognize Σeiei as the unit tensor. So we have:



which allows us to conclude:



We can go ahead and do some work on this integral. So first, the angular integration will just give us 4π. So then we’re left with:



We can change variables to get the T-dependence out,



If we need to know the value of that integral, we can use the Bose function thing defined in the Stat Mech/Boson gas file:



and then say,



So for low T (b/c we used the low energy approximation to the excitations), we have:



This matches experimental measurement of nN, shown below:



**Heat Capacity at low T**

Now let’s work out the heat capacity at low T’s. We pretty much already did in the Free Day folder when we did phonons. But we’ll adapt the calculation. So at low T, we know the excitations are of form ω = kvs, so:



And then we’ll borrow the boson function definition (see Stat Mech folder free boson file thing),



In which case, this comes to:



And now the specific heat is just a derivative away:



so we have:

