**Excitations**

**Particle-Hole Excitations**

Well, at least for ES’s close to the GS, they’re those quasi-particle excitations, npσ, in terms of which we have the system energy as:



The energy of a single quasi-particle could be written as εpσ = δE/δnpσ, which is how much the energy of the system changes when we add the particle. And this works out to:



(but don’t expect ε to actually depend on σ, unless we have a magnetic field around I suppose) Generally, npσ is specified in some fashion (though often in terms of εpσ itself), and from the quasi-particle energy equation we can solve for εpσ. We’ll find it’s necessary to have an expression for the ‘single quasi-particle’ density of states ρ(εpσ). It seems that this would not be a well-defined thing in general, as it depends on npσ. But if we restrict ourselves to a given configuration npσ, then this would fix all the single quasi-particle energies εpσ, and then we could construct from this a ρ(εpσ). Though in general it seems ρ(εpσ­) would be npσ dependent, I think the only ρ we need to know, is the one when the quasi-particles are in the ground state configuration, or near it. And we only need it for energies close to εF\* (I guess we only *can* know it only for energies close to εF\* as our formula for εpσ presumes pσ is close to the Fermi surface). So near this configuration, δnpσ = 0 basically. And so εpσ = εpσ(0) = εF\* + vF\*(p-pF), which is just what the free spectrum is, close to the Fermi surface, but with a renormalized velocity (or equivalently, mass). And so we’d get:



**Special case: isotropic thermal distribution**

We can calculate the quasi-particle energies for a special cases. First, let’s say npσ is isotropic, and moreover, given by the Fermi function, nF(εkσ) = 1/(eβ(ε\_kσ-μ) + 1) so that:



and the deviations δn are taken from the ground state as usual,



Then we have an implicit equation to solve for the energies.



Now consider f:



We can see that upon substitution of f into our energy equation, and summing a spherically symmetric distribution function against it, the guys with cosθ´ in them will be eliminated. So now we have:



Well μ increases as T2, so to first order we can say μ = εF\*. And to first order in T, we can treat density of states at the Fermi surface as a constant ρF\*. Then since the Fermi distribution function is odd about the chemical potential, this integral goes to zero too. So we find that for isotropic Fermi distributions, to first order in T, we can say εpσ = εpσ(0).



So there. Note that just because our energy is εpσ(0), that doesn’t mean that the interactions didn’t affect the energy spectrum, ‘cause the interactions are reflected in the renormalized values εF\* and vF\*.

**Special case: isotropic thermal distribution in magnetic field**

Now say we have particles in a magnetic field. The magnetic field will couple to an electron spin, with an effective magnetic moment, g. So,



and again we’ll say that npσ is given by the Fermi function. Recall the deviations δnpσ are taken from the *non-polarized* ground state, so,



So we have an implicit equation to solve for the energy levels,



Now gonnna try to solve for εkσ in the low T and H limit. Well μ depends on T to second order, and μ can only depend on H to second order too (there is no preferred vector in the system, except for H, and so can only depend on **H**·**H** = H2). So we can treat μ as a constant εF\*. When we plug our f:



into the equation, we can see that once again the sum over the cosθ´ terms will give us zero, as we should have that εpσ is isotropic. So now:



But this time, the sum over p´ doesn’t give us zero, because we aren’t evaluating εp´σ´ at H = 0 in the Fermi function, and so the nF(εkσ)-θ(εkσ(H=0)-εF\*) isn’t odd about the μ (= εF\*). So let’s expand the Fermi function in powers of H.



where those two terms in the bracket go away because they are together odd about μ = εF\* and so the sum over p´ gives zero, just as in the previous problem. I think we’ll do chain rule on the derivative,



Now let’s take the T → 0 limit, and we’ll have:



Gonna take a guess and suppose that ∂εpσ/∂H doesn’t depend on p. This will have to be self-consistently verified later. But the fact that it doesn’t in the free particle case at least provides some motivation for it. Once we pull that out, then the (1/V)Σp´δ(εp´σ’-μ)|H=0 term will give us the density of states (or really, half of it, since no spin sum yet). So now we have:



Now we’re implicitly only trying to work εpσ out to first order in H, so we might as well say so on the LHS. So expanding the LHS out to first order in H, we have:



Now εpσ(H=0) is just what we found in the previous problem, namely εpσ(0)(H=0) = εF\* + vF\*(p-pF). So,



Well I guess I’m going to have to propose an ansatz. So let’s say,



Then,



and so,



So good guess. Then,



And we do see it’s independent of p, verifying an earlier supposition. So altogether, we have:



And so finally,



Of course, this line could’ve been anticipated earlier, since we had already solved for ∂εpσ/∂H. But anyway, we see that all this just renormalizes the effective dipole moment. So in both our problems, we see that εpσ is εpσ(0) but with possibly renormalized coefficients.

**Special case: isotropic constant distribution**

Last one. So before, we’ve been looking at the energy levels for the δnkσ given by nF(εkσ)-θ(εkσ < εF\*). Now let’s look at it for δnkσ = θ(kF < k < F), which is to say, we’re just adding particles to the ground state, to create a new ground state configuration with Fermi wave vector F. So what are the energies of these new quasi-particles? We have:



As usual, the sum over p´ will vanish for the F1s,a terms. And then the spin sum will vanish too.



Keeping in mind that these p´ states label the free particle states as well, we know that this sum can be done in the free-particle state frame of mind. So, this is:



Now of course



So this works out to:



Filling in εp(0) = εF\* + vF\*(p-pF), we come to:



So there. It just adds a constant energy.

**Collective Modes**

Since there are short ranged interactions between electrons, I think we do have acoustic collective oscillation modes in the Fermi liquid. We already saw this to be the case in the Nearly Free Electron Interaction folder.