**Thermal Equilibrium Properties**

We’ll take a look at a variety of thermal properties…

**Magnetic Susceptibility**

Now let’s look at the magnetic susceptibility. We’ll recall that it is defined via:



The non-equilibrium paramagnetic susceptibility is calculated in the non-equilibrium folder. If we take the limit that ω → 0 (and q → 0 too, why not?), we’ll get the constant field equilibrium susceptibility. This is:



But this is just the non-interacting result. So nothing special. And it plainly exhiibits paramagnetism.

**Ferromagnetism ??**

The Coulomb interaction itself suggests a tendency for the e- gas to exhibit Ferromagnetism. When the Coulomb interaction between conduction electrons leads to magnetic properties this is called *itinerant exchange*. In this case, it happens because of the renormalized single particle spectrum. Recall that we found under the HF approximation, the following energy spectrum:



which looked like the blue guy,

Chart, line chart

Description automatically generated

And also found the ground state energy,



which looks something like this, as a function of kF (but note kF depends on N too)

Chart

Description automatically generated

Our formula for EGS presumed that we filled every momentum state with both spin up and down particles as we went along. But apparently (so says Ashcroft & Mermin), if we go back and analyze our calculation, then we’d find the spin up and spin down electrons kind of decouple, and we can specify any number N↓ and N↑ spins in their own separate ground state configurations. And we’d get the same formula for the energy too. So we would then be able to say:



such that N = N↑ + N↓. In the old configuration, we had:



and so,



as expected. But now let’s try:



(cause if we’re filling all spin up, then we’re taking up twice as many k-states, which means twice the volume of k-states, which means the radius of the Fermi sphere must increase by factor 21/3 owing to V = (4π/3)kF3. So this energy would be:



So when will the new energy be lower than the old?



In our units, the Bohr radius is a0 = 1/me2. So can write this as:



Recalling kF = (3π2n)1/3, this equation predicts Ferromagnetism at low densities. But it turns out that if we replace the Coulomb potential with the screened Coulomb potential, then the same analysis reveals Ferromagnetism at high densities rather. Moral of the story, though, is that the nearly free electron model probably does not apply to electrons responsible for magnetic behavior. And I think more serious calculations lead to the conclusion that nearly free electron (such as those in the s and p subshells) do not support Ferromagnetism. But in the tight binding model, we find there is support for ferromagnetism/antiferromagnetism, stemming from d-orbital electrons.