**Electric Susceptibility**

Yeah. So we’ll do this a non-GF way.

**1st quantization way**

So one way to get the time-dependent susceptibility is from the time-dependent induced charge density. We can get this, at least to first order, using the formula (see Stat Mech folder/Kubo Formula)



where A = Σiai is a many body operator representing some physical quantity which is the sum of single particle operators, *a*. And |ψn(t)> are the time-dependent single particle wavefunctions. So then we can say for the electron charge density,



So far this is exactly how we got the time-independent ρ(r). The induced charge density would be:



Now let’s look at the first order correction to the wavefunction (spin should be a spectator variable here). We’ll turn the perturbation on adiabatically, so we’ll have something like V(r,t) = eφ(r,t)eηt. But then note we’ll also be adding a blue exponential factor, consistent with how we treat the time-developed quantum distribution function (see Stat Mech/Kubo Formula),



This is necessary so that we eliminate the ephemeral effects of turning the perturbation on suddenly, and isolate the steady state response. We’ll also use the phase convention associated with infinitely timed potentials (see Quantum Mechanics/Time-Dependent/Interaction Picture file), whereby the system’s time evolution in the absence of a potential reduces to what it would look like if evolved at time t = 0. Finally, our potential includes contributions from charges in the system. So it is the total potential. Need to work out what the first order correction looks like in position space,



I guess we can fill in the unperturbed wavefunction ψkσ(r) = eikr/√V (again, spin is just a spectator),



Okay so now forming the density,



Can extend time integration to +∞, by writing,



Now this takes the form of a convolution, and we can say, taking the Fourier transform of both sides (maybe see EM folder/Metal TD for an example? or can see Stat Mech/NESM Quantum file for simpler way perhaps),



These separate FT’s are:



So filling these in,



Now need to get the second guy,



It’s just the complex conjugate of the first guy, in real space. So,



Taking the Fourier transform, we’ll find:



So adding these together,



Let’s change variables in second one to k → k + q. Then, we have:



Then doing the trival spin sum, we have:



which tells us that:



Reassuringly, this matches the ω = 0 Lindhardt function. Here’s another way,

**2nd quantization way**

Now let’s move on to another way to do this. There are many ways to work this out – see NESM (Kubo) file. We’ll opt for the approach where we calculate the thermal average of the time evolution of the density operator (all from Mahan basically). So we’ll start with:



and begin by writing out the Hamiltonian for ‘free particles’ nonetheless moving under the influence of an electric field/potential generated by both external (free) charges and themselves (bound).



Then we form the induced electron density function:



Taking the derivative of the density operator w/r to time,



So that latter one is:



So putting this result in,



We can make a change of variables in the latter sum to write,



We could take the < > of both sides, but our result wouldn’t be self-consistent because we don’t know what <ck-q,H†(t)ck,H(t)> is. So instead what one typically does is go back to just this term alone, ρkσ(q) = eck-q,σ†ckσ, consider *its* evolution equation, and make some approximations to make it self-consistent. We’ve already done the necessary work – just have to take out the sum over k. So consider:



Now the random phase approximation is to presume that q´ = -q gives the largest contribution. So we have:



Now take Fourier transform, and presume n’s are constant (think we can do this to first order in the perturbation at least – see NESM file (Kubo).



*Now* if we sum over kσ we have (n’s don’t depend on σ),



Now we take the thermal average of both sides,



So we have [think it’s acceptable to replace ρ with ρind at this point because taking the derivative of ρ would’ve eliminated the background charge density, or can say that ρind is what’s proportional to φ(q,ω), and the background jelly charge is just a constant all by itself],



Well instead of -ω, I should have ω + iη. Have to be more careful next time. The negative sign probably doesn’t matter at least. Whatever.