**pn Junction Excitations**

We’re going to consider the p-n junction. For point of reference, it’s useful to have the following values in mind, for Si, say, which could be doped with Al, or P, to make it a p-type or n-type semiconductor.



In the last line we’re assuming ΔN = Nd or ΔN = Na, i.e., that the p or n type semiconductor is basically a pure p or n type, and also keep in mind these rough energy scales,



First we’ll examine the equilibrium charge distribution of a pn junction, and potential difference that arises from the contact junction in the absence of an external potential difference. So let’s consider a p-type semiconductor joined to an n-type semiconductor. The p could be GeGa; the n could be GeAs. The electron levels and the chemical potential is illustrated below. The chemical potential, as we saw in the impurities folder should still be roughly half-way inbetween the gap for both of them, separated by O(0.1eV), with μn-type > μp-type. So the difference between them has been exaggerated. In the depictions below, I’m going to draw the electron/hole distributions to first order, assuming ΔN >> ni, skipping the O(ni2/ΔN) contribution we talked about in the Thermal Equilibrium Properties file in the previous folder up.

Chart, shape

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Though each side is neutral, we have more electrons on the right than on the left (maybe it’s more important that there are more *mobile* electrons on the right). And the chemical potential on the right is higher. So the electrons will want to diffuse to the left, until the chemical potentials are equal. But this will induce a net dipole moment across the interface, and therefore an electric potential difference. In drawing below, three electrons on right migrate to the left. As a consequence of the potential difference, φ, induced, the energy levels of the electrons will be altered. We’ll use a semi-classical approximation, and just say that the energy levels are altered by an amount equal to the local potential difference, so ε → ε + eφ(x) (and e is negative). So ε = ε – |e|φ(x). And since φ(x) is negative on the left, and positive on the right, the energy levels will go up on the left, and down on the right. But near the interface, as φ(x) → 0, they will be more or less unmoved, as φ(x) is small. So the donor and acceptor levels will get close together far from the interface, but practically unmoved close to the interface. BTW, I used the μ line as the reference line φ(x) = 0 for the induced electric potential.

Chart, histogram

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So we’d like to see how the populations of the levels and bands changes, now. And we’d also like to know what φ(x) is. We’ll presume that φ(x) is weak enough that the levels are still pretty far from the chemical potential, and we can still use Maxwellian statistics. We can self-consistently use Maxwell’s equation to determine these. So given φ(x), from equations found in the Thermal Equilibrium Properties file, the populations of the donor and acceptor levels is (note each of these formulas is basically degeneracy of the energy level × Maxwell factor):



and those of the conduction and valence bands, is:



Again, e is negative. And note the law of Mass Action still holds: ncpv = ni2. We need an expression for the local charge density. It helps to look at the first diagram above. In a neutral p-type guy, the number of holes in the acceptor level and valence level adds up to Na. This is because with a filled valence band, there are Na holes in acceptor level ‘band’ (each level is singly occupied), and then if a hole drops from acceptor level, it will go to valence band – so holes conserved. And this is a zero net-charge situation. A net charge density of ρp = -e(pa + pv – Na) will be present if they don’t equal (e is negative). Parenthetically, can see we are neglecting the possibility that electrons are in the conduction band (perhaps by jumping up from the valence band). So we are therefore neglecting the ni2/ΔN contribution to electron density in the conduction band, and to the extra hole density in the valence band. Likewise, on the right, a neutral n-type guy will have Nd electrons in the donor level ‘band’ (each level is singly occupied), and then if an electron jumps off the donor ‘band’ it will go into the conduction band. So electrons are conserved. And so the number of electrons in the donor and conduction bands equals Nd. If they don’t equal, there is a net charge of ρn = e(nd + nc – Nc). And parenthetically, can see we are neglecting the possibility that holes are in the valence band (perhaps due to electrons jumping up from the valence band into the conduction band). So we are therefore neglecting the ni2/ΔN contribution to the hole density in the valence band, and to the extra electron density in the conduction band. Adding the two together gives us the net charge density across the board. If we extend the definition of Na,d to be x-dependent:



we can say,



They say we’ll assume the acceptor/donor levels are completely ionized, so that nd(x) = pa(x) = 0. As you’ll recall, we found this to be a good approximation in the Thermal Equilibrium properties file in the previous folder when the number of extrinsic carriers is much larger than the number of intrinsic carriers. So now we have:



The charge density is related to the potential via Maxwell’s equation (in faux Gaussian units):



We also have boundary conditions. We will presume that far from the interface, the semiconductors act like their normal selves: the donor level electrons are all up in the conduction band, and the acceptor level holes are all down in the valence band. So,



With these boxed equations, and the generic formulas for the carrier densities nc, pv, we can solve for the potential, charge density, and densities. Before we do, let’s consider a few useful corrolaries. From the boundary conditions and the formulas for nc(x,T) and pv(x,T), we can write the net potential difference across the two semiconductors in terms of the carrier densities. Equating the products of both sides,



So we have a handy relation for Δφ = φ(∞) – φ(-∞):



For Si, we found that Nc ~ Pv ~ 1025 carriers/m3 and Nd, Na ~ 1022 carriers/m3. So Δφ ~ |e|-1[Eg + kBTln(10-3)] ~ |e|-1Eg, as Eg ~ eV, and kBT ~ 0.03eV. This tells us that eΔφ = e[φ(∞) – φ(-∞)] is on the order of the energy gap Eg >> kBT. And this also says that Δφ itself is on the order of volts. Using the same boxed boundary condition statements, we can write nc and pv without direct recourse to the chemical potential:



and,



So,



We can approximately solve the Poisson equation by approximating the charge density profile. Let’s fill these results into ρ(x).



So can see that when |x| → ∞, we have (remember e is negative):



since |e|Δφ ~ Eg >> kBT. But for x close to the interface, recalling e is negative, we have:



So it kind of looks like the picture we drew up above, where all the donators that were on the right jump to the left, filling in the holes present. And likewise, all the holes that were on the left, jump to the right. Introducing the to-be determined cut-offs for the asymptotic behavior, -dp, and dn, we can approximate the charge density as:



This equation basically says that all the electrons in the n-type drop out of the conduction and donor bands (an empty conduction and donor band would mean net charge (density) -eNd) and fill up the valence and acceptor bands (a full valence and acceptor band would mean a net charge (density) of eNa). Charge conservation will require Nadp = Nddn. Parenthetically, do note that despite the fact that there is excess/net charge only within the region -dp < x < dn, there is *mobile* charge only *outside* this region. This is because inside the region -dp < x < dn, the valence band is full, and the conduction band empty. So we can get no net current here. Thus, this region is called the ***depletion zone***. Can see 2nd picture at top of page, or I’ll just draw it again here.

Diagram

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One could also say that this is why we have an electric field in this region – because the charges are immobile and therefore cannot screen the field away, like they can outside this region where the potential is constant/field is zero. But anyway, filling this into Maxwell’s equation, we have:



So we just need to integrate this twice and satisfy the boundary conditions. Should have constants at the extremities and some 2nd order polynomial in between. So we could have something like,



Continuity and differentiability at x = 0 requires,



and,



This latter condition is a statement of overall charge neutrality. Putting this into the former equation, we have:



Solving for the other one, and altogether, we have:



And we’ll recall the formula for Δφ as well up above. Filling in typical Na,d ~ 1/108 atom, and Δφ ~ eV, and ε ~ 10, we get lengths of around 1nm or more. So now we have the potential profile, and the accumulated charge density, as well as the new energy levels incorporating this potential.

**What if we don’t neglect the ni2/|ΔN| contribution to electron, hole carrier densities?**

So recall some of the blue highlighted passages above where we said we were neglecting the extra ni2/|ΔN| contribution to the electron density/hole density in the p-type’s conduction/valence band, as well as the extra ni2/|ΔN| contribution to the electron/hole density in the n-type’s conduction/valence band. I’m not going to mathematically work through the implications of undoing that assumption. The upshot is something like this:

Chart, diagram

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where the Na + ni2/Na ≈ Na holes in P’s valence band rapidly diminishes over the course of the depletion zone to the ni2/Nd holes in N’s valence band. And the Nd + ni2/Nd ≈ Nd electrons in N’s conduction band rapidly diminishes over the course of the depletion zone to the ni2/Na electrons in P’s conduction band. ni2/|ΔN| << |ΔN| of course, but the picture doesn’t do this inequality justice, so it looks like there’s a lot more electrons/holes in the depletion zone than there really are. We will observe, that inside the depletion zone, there is a net charge density, and a consequent dipole moment. But outside the depletion zone, everything is neutral.