**Bragg-Williams Inhomogeneous model**

Now we’re going to take a different approach that will allow us to study the influence of non-uniform magnetic fields within our magnet. Let’s consider again consider our Hamiltonian in an inhomogeneous field:



J is an N by N matrix whose elements give the interaction strength between the i, j elements. For instance, if we were dealing with a 1D line of spins, and z = 4, then an ‘isotropic’ J would look like,



and hi can also vary with index/position.

**Canonical Transformation to and calculation of F\*(T,m(x))**

Since we might have noticed that the free energy can be written entirely in terms of the magnetization, it kind of makes sense that a more natural Free Energy would be the one whose canonical variables are temperature and magnetization, rather than temperature and magnetic field. We can do this via a Legendre transformation. So let’s define F\* = F + Σimihi = F + a-d∫ddx m(x)h(x). Then F\* = F + a-d∫ddx m(x)h(x). Then dF\* = -SdT – a-d∫m(x)δh(x)ddx + a-d∫δm(x)h(x)ddx + a-d∫m(x)δh(x)ddx = -SdT + a-d∫δm(x)h(x)ddx. And then F\* itself is F\* = U – TS + a-d∫m(x)h(x)ddx = Uexchange – a-d∫ddx h(x)m(x) - TS + a-d∫ddx m(x)h(x) = Uexchange – TS. (exchange stands for the exchange interaction). So replacing the local spin with its local average, we have for Uexchange,



And from our work in the homogeneous Bragg-Williams file, we can surmise that our entropy will go to:



So our free energy should be:



**Going to continuum limit and keeping only long wavelength (small k) terms**

We can go to the continuum limit. Employing the same reasoning as in the last file, we’ll end up with (guess we’re ignoring that 2 again):



**Small m expansion near the critical point**

And we can expand our result for small m,



So then,



Like last time in the homogeneous BW file, we can define,



and then we have:



where this time m(x) is not an implicit function of the field/temperature, but rather an independent variable. A functional derivative would give us the equation of state:



So we have the same equation of state as before (at least in the T → Tc limit):



Again we can define the additional susceptibility/correlation length guys.



We can get an equation for χ by taking a functional derivative of our m(x) equation w/r to h(x). This will give us what we had before. Or, can formally do it like this:



Inverse has to be understood in the matrixy sense that:



Evaluating this expression we have:



And can invert this relationship,



And so finally, like we had before:



**Appendix**

Let’s check that our Weiss and BW Free energies match. So we have:



and our equation of state,



and we’re supposed to have:



So that works out!