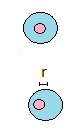
**Thermal Properties**

Now let’s consider the thermal properties of these electrons.

**EM Susceptibility**

In the EM folder we examined the electric susceptibility via a purely phenomenological model. We presumed the electrons, being tightly bound, would simply oscillate like a HO, in the presence of an oscillating Electric field. Recall our model of an atom as a positive nucleus surrounded by an electron cloud.



and recall we worked out from Gauss’s law that if we displaced the cloud to the left a distance r, the nucleus would exert a restoring force:



where e is the (absolute value of the) cloud charge, k = 1/4πε0, and R the radius of the cloud. So in the absence of any external field, the electron cloud would obey the equation:



where m is the mass of the electron. Now say we place this in an external field, potentially oscillating, E(r,t). Then our equation becomes:



(we may ignore the r dependence of E because the cloud will be so small that E cannot very hardly at all over its length. And now we will attempt to model dissipative forces.

Damping of the oscillations occurs because of energy lost by the oscillator due to bumping in with other objects, radiation (oscillating dipoles radiate energy…) and also because of frictional/drag forces exerted on it by nearby atoms, or even inexactness of the quantum wavefunction. I don’t know the physical mechanism of dissipation. Normally I wouldn’t include dissipation here in the ‘free file’, but I guess I will because I don’t know whence it comes. Including these damping terms requires a term proportional to the speed of the oscillator (but isn’t the energy lost due to radiation proportional to ω4 – yes but we’re using the simplest model). So our equation would be:



We can solve this equation by taking the FT of both sides:



Well, now, the polarization (density) is:



where n is the number of dipoles/molecules per unit volume. And so the Fourier transform is:



So it follows that the polarization is:



and there upon, the susceptibility is:



We can define the plasma frequency to write:



Plasma frequency is around infrared I think, for insulators. Note that for ω << ω0, χe(ω) would be roughly constant, and then we could go back to the static approximation. We can write the dielectric ‘constant’ as:



Generalizing to the possibility of multiple resonant frequencies (determinable from QM), we can write,



where fj is the strength of the resonance at ωj. And of course



Not sure how T would play a role in this, it seems there ought to be an exp(-βωj) factor in the summand. But whatever.

**Magnetic Susceptibility**

For the magnetic susceptibility, we made a more precise calculation. Recall its definition:



In the SM folder we started with the Hamiltonian (it seems we could easily modify this to address the electric susceptibility above, but another time…):



And recall this can be written as:



Using Hund’s rules, the lowest lying energy states are:



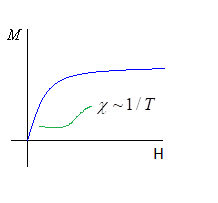
where



Presuming JT ≠ 0 [shell not full or ½ full -1], there will be JzT values, and the substance will have a paramagnetic susceptibility, which ought to dominate over the diamagnetic susceptibility, at least in the small B limit. That’s all I’ve calculated. And it was:



And plotting this we get,



Well I guess M isn’t exactly a susceptibility. So we’d have to differentiate this w/r to B. Looks like it’s linear, and so we’d get a constant susceptibility for our constant field, at least for low B/H. We should be able to get the diamagnetic susceptibility too, the same way, but I haven’t.

**Heat Capacity**

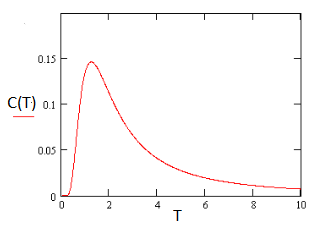
I don’t know how the energy levels depend on JT per seʹ, so I can’t exactly calculate the heat capacity. But I would presume some finite # of energy levels, which would give us something like:



Then the heat capacity would follow:



A typical plot is:



We can make sense of this plot in that C(T) is zero for low T since T isn’t high enough for the levels to be thermally populated. As T increases, the levels can be populated and U increases with T. As T get’s larger though, the population of E2 saturates to 100% and higher T’s cannot increase the internal energy any more. So C goes back down. Now

**Absorption**

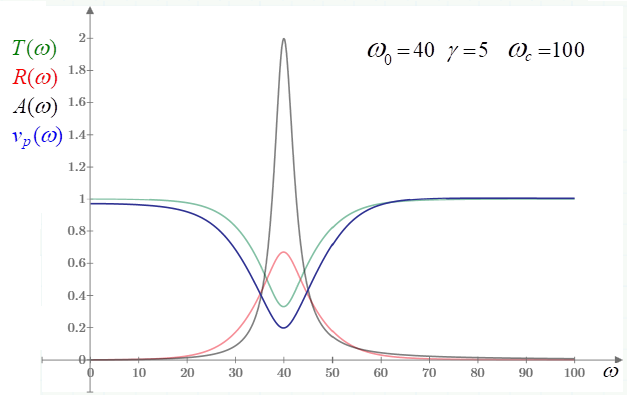
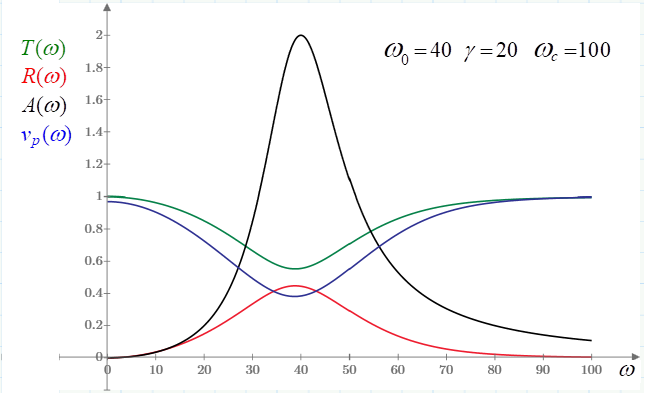
The absorption spectrum was plausibly argued to simply be:



Given our expression for ε(ω) we end up with:



And so we’ll get something like (you’ll recall):

Again, for smaller frequencies, we will expect absorption, but larger frequencies will be harder for the e- to follow, and so will get transmission. The mechanism for dissipation (γ) could be impurity scattering (?), or e-e interactions (?), or phonon emission.