**Interaction**

See page 32 (Mahan) Piezo-electric interaction.

Also check out the Metal e-ph interaction file. There’s a section on e-optical phonon interactions, that is probably relevant here. Oh well, I see it is down below already.

We’ll use a tight-binding model to characterize the insulator electron-phonon interaction. But there will be no hopping term as the electrons are fixed on the lattice sites. So we have something like this:



Note this problem is exactly solvable. A more accurate model would take account of the electrons’ oscillations between orbitals, i.e., would take account of some polarization current. FWIW, I think the coupling of the polarization current to the EM photon field is done in the Light/Matter file.

**Mobile e-‘s interacting with optical phonons (Mahan) (this was originally in corresponding metal file)**

*Now consider that if there is a basis, we can get optical phonons.* Here we consider coupling between electrons and optical phonons. This sort of interaction would be dominant in insulators (where there aren’t many other mobile e to screen the phonon modes). Optical modes of oscillation result in ions in the basis (or ions on the lattice) oscillating out of phase. For instance we can have two atoms in the basis oscillating back and forth from each other – 180 degrees out of phase. I don’t think this would describe acoustic like oscillations because there the atoms are oscillating in phase, and they therefore move by close to uniform amounts. But a uniform displacement of the lattice atoms would look just like the atoms that were undisplaced – we’d still have a uniform background charge (jelly electrons) beneath a uniform lattice charge. So no net E-field produced by incongruities in the charge density.

Consider that the mobile e- set up to first approximation a uniform jelly charge which cancels out the + charge of each ion. So we have a net 0 charge. Against this backdrop, we have the ions oscillating back and forth. These oscillations set up local periodic polarization of the charge background, similar to what we have in polarizable mediums, but only this time it isn’t the electron clouds’ deviation from the ion that sets up the polarization, but the local deviations of the positions of the basis atoms (note the basis atoms’ deviations from their equilibrium positions is described by plane waves with some k in BZ, so it is periodic). This polarization in the charge density creates an electric field in the material which will interact with the mobile e-‘s. So the electric field set up by the oscillating atoms comes from the polarization and E’s coming from P’s are described in EM 2 via [again, fake Gaussian units here]:



Thus the induced E-field, coming from the basis atoms, that interacts with the electrons is proportional to the polarization. And the polarization is proportional to **U**(r). So we have,



where a3 is the unit cell volume. Now,



(where we’ve kind of taken a limit that R → r so we have a continuous U field) And so,



Then we have:



We may determine a potential from this. Consider that:



and then comparing, we have for φ,



And now we integrate this times the e-density over all space and obtain the e-ph Hamiltonian. We would find:



If we say that a3 = V/N, then we can write this as:



which is the same as what we found above, basically.