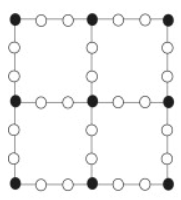
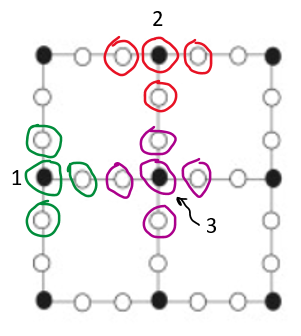
**Electron-Electron Interaction**

Here’s another model. I’m not sure what it’s physical motivation is. But consider a lattice,



On each lattice point (dark filled in circle), there are Ising spins, σi. And on each bond there are two s-orbitals (empty circles). The pair of s-orbitals is called a dimer. The dimer can hold, therefore, up to 4 electrons. The Hamiltonian of the system (or at least the interesting part?) tabulates the energy of these dimers. There are two parts of the Hamiltonian, a kinetic/hopping term for the electrons jumping between the two sites within the dimer, and a potential energy/interaction term of the dimer which comes from the interaction of each site within the dimer with the lattice point nearest it.



So for instance, consider the dimer between lattice points <13>. The s-orbital (green) nearest lattice point 1 would be designated 1 in this case, and the s-orbital (purple) nearest lattice point 3 would be designated 3. And the Hamiltonian for this dimer would be:



If we considered instead the Hamiltonian for the dimer between lattice points <23>, we designate the red s-orbital as 2, and the purple s-orbital as 3. And H23 would be:



And so the entire Hamiltonian would just be a sum over all bonds, which amounts to a sum over all nearest neighbor lattice points:



Maybe the interactions between electrons are implicitly included via the Ising term. For instance, all purple s-orbitals would interact with each other, implicitly, because they interact with the third lattice site? Will note that if you tried to integrate out the Ising sites, then because of the ∫dx exp{-ax2 + bx) = √(4π/a)exp(b2/4a) identity thing, it looks like this would transform the independent dimer-lattice site interaction (bx) into a coupling between dimers (b2), right? Anyway, apparently this model can be solved exactly – it can be transformed into an Ising model. And it exhibits phase transitions and things, including antiferromagnetic and ferromagnetic transitions. I think the latter occurs at half-filling (2 electrons per dimer). And note, because of particle-hole symmetry, the behavior for 0 < n < 2 filling is the same as the behavior for 2 < n < 4 filling.