**Interaction**

The FL formalism can describe well the 3He gas-liquid up the phase transition point (approaching from the right), but a better model is needed to describe superfluid state.

Diagram

Description automatically generated

Basically, we need to better describe interactions between quasi-particles. To make progress, it’d be nice to equate the FLT formalism to a 2nd quantized Hamiltonian in terms of quasi-particle creation/annihilation operators (we already know what it is in terms of free particle operators, but in terms of those, the interaction is too large to treat perturbatively). This is especially so since we succesfully analyzed a similar fermionic superfluid transition – i.e. superconductivity – in such a format. So we’d like to say something like:



where cpσ annihilates quasiparticles, the ‘free’ quasi-particle spectrum εp(0) is:



and Mσσ´(q) is a spin-dependent interaction (see 2nd quantization in Quantum Mechanics folder for spin-dependent interaction potential stuff) which models their interaction somehow, i.e., is somehow related to the fpσ;p´σ´ guy. Such a description seems possible in principle? If we were to agree that the quasi-particle states were some linear combination free particle states, then we could make a unitary transformation of the free c’s into the quasi-particle c’s. And Mσ´σ(q) would be that unitary transformation on V(q), or related to it at least. Anyway, it seems that people just treat M phenomenologically. Like with the BCS interaction, we presume the 3He’s will hook up in opposite *momentum* Cooper pairs. So,



where in the last line we changed variables q → k´-k, and then k ↔ k´. At this point we would probably use a phenomenological model for M based off the f guy:



Plugging in typical values,

A picture containing table

Description automatically generated

We see this comes to:



So we see a slight (relative) attractive interaction for spins oriented in the same direction. When the quasi-particle interactions are handled more rigorously, it seems that this attractive interaction can actually become quite sizeable, and larger than the first term (i.e. one w/o the σσ´). In so far as we’re interested in the effects stemming from the effective attractive interaction, I think we just say:



where ·´ = cosθ. Of course this phenomenological Fermi liquid model means we’re implicitly restricting k and k´ to the Fermi surface. And in a more general treatment, there’d be some sort of spin term attached. It seems though, that when I naively stick a σσ´ onto the Mσσ´(k,k´) term, the solutions that should work out don’t work out. So I guess a σσ´ spin-interaction is too naïve. Perhaps it should be **σ**·**σ**´, where these are the Pauli spin matrices. But then we’d have to work out the 2nd quantized version of a two-particle interaction that, in 1st quantization, looks like:



where and **´** are vector momentum operators, p and p´ are their magnitudes, which we can just take as the Fermi momentum, and and ´ are the usual vector spin operators. This form makes it clear that we basically have an effective exchange interaction going on. Acting on some two particle ket |kσ>|k´σ´>, would return the potential energy of the pair. But yeah, I’m not going to work that out, ‘cause it will make the forthcoming analysis quite a bit harder.