**Non-Equilibrium Properties**

So now we’ll run through a calculation of the absorbtivity and DC conductivity. Recall we had,



and,



and also from GF file,



where we take Δ to be real, and recall the Pauli spin matrices,



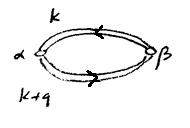
and **1** is the unit tensor, which I’ll leave off. We’ll also use, later, the spectral representation of **G**,



Assuming Δ is real still.

**General formula for absorbtivity Reσ(q,ω)**

So let’s work Reσ(q,ω) out. Copying, somewhat, what we did in the Metals/Impurities/Nonequilibrium/Conduction/Drude file. So we have, basically,



where the two GF’s are the fully interacting GF’s. So,



where,



To evaluate S, we have to do that Matsubara sum over the frequencies. So recall the general technique elaborated on in the Stat Mech Math Appendix,



Recalling our G:



we see it’s analytic, except at the poles of course, so we can just say,



Instead of filling in our G though, we’re going to use the spectral representation of G instead, and then do the sum over residues that way. So the complex frequency (i.e., analytically continued) GF is:



and **A**k(x) is the spectral function. And so we can say,



recalling nF(x) is periodic in boson frequencies. Now let’s analytically continue, and go back to Re**σ**(q,ω):



And now use 1/(x+i0+) = P(1/x) – πiδ(x). So,



Presuming we have an isotropic medium, we have σαβ = σδαβ, where σ would necessarily be (1/3)Tr[σαβ]. So let’s assume so, and then,



and so,



We might profitably compare this to what we got, by a slightly different handling of the poles/branch cuts in the Metals/Impurities/Non-equilibrium/Conduction/Drude file. It’s mathematically the same.

**Absorbtivity Reσ(q=0,ω>0) @ T=0**

Now let’s focus on the absorbtivity. A free metal has zero absorbtivity (at q = 0, T = 0) except at ω = 0. So we might expect something like that. Certainly, we’d like to see that it’s zero for ω less than 2Δ, when T = 0. So as T → 0, the nF(x) – nF(x+ω) term becomes just 1 within window x ∈ (-ω, 0). So can say,



and we’ll recall,



Now let’s fill this in. Note Tr(σiσj) = 2δij (easy to show, or see Relativistic QM folder for instance). So



So to have a non-zero integral, the δ functions require:



Can solve for x in top two:



The inequality requires we choose -sgn in first line, and +sgn in second line.



Equating the two gives us:



FWIW, we can simplify the integral a bit now, choosing the appropriate roots in the δ functions, and using δ(f(x)-f(a))=δ(x-a)/f´(x),



where in the penultimate line we take advantage of the δ function to change the integrand somewhat. Let’s specialize to q = 0.



So we get 0!



(and Mahan confirms this does in fact go to zero – technically need impurities to give us non-zero absorption; but consider case of absorption in semiconductors where we found non-zero result because of energy gap, but no impurities…hmmmm) This is in fact the same as we found for ideal conductors when we analyzed the absorbtion of independent electrons in the presence of impurities and took the τ → ∞ limit. Well, more precisely, we found,



So we shouldn’t have found 0 for ω = 0. So let’s see if we can do ω = 0 better.

**Absorbtivity Reσ(q=0,ω=0) @ T>0**

Let’s take the ω → 0 limit and see if we can get the δ function. Could go back to:



and fill in Ak(x),



And we get:



sgn(x)sgn(x+ω) is 1 for x > 0, -1 for -ω<x<0, and 1 for x < -ω. Since ω → 0, I think we can just say it’s 1 for all x. I’m going to go to q = 0 now,



and break down the δ functions,



Now,



Guess I’ll take ω → 0 limit in the first bracket of terms, and we just get ½. And also take ω → 0 limit in the nF(x) braceket. Then apply the first set of δ’s to the second set,



and now…well if ω → 0, then we shouldn’t be able to satisfy ω = ±2√(ξ´2k + Δk2), except where that’s zero, which would be strictly at the Fermi surface. But that shouldn’t account for any volume in k-space. So just have:



‘cause derivative is an even function. Then can say that since nF is strongly peaked about 0, this makes ξ´k effectively zero and k2 = 2m(ξ´k + μ´) effectively μ´ = εF. Then also replacing d3k integral with one over (spinless) density of states, we have:



Now since n´F­ is peaked about 0, restricting ξ´ to roughly μ´, as we said, this restricts the density of states to its value at the Fermi surface roughly.



So we can say,



Now fill in ρF = 3n/4εF,



where we (re)introduce the definition from the Meisner effect file for the ‘normal’, as opposed to ‘superconducting’, electron density.



I’m not sure this is correct though. Maybe we should have n, or ns(T) in the numerator? I’m told we should have ns(T). Maybe if we go back and handle the singularities differently, like we did in the Meisner file, we can get the missing piece. But I’m going to use voodoo mathematics….so recall we have:



And we worked out,



But we have problems with the ω = 0 limit here, in σ, due to the 1/ω multiplying **K**.



And so,



I don’t believe there is (at least there isn’t supposed to be) an imaginary part to σ in the q = ω = 0 limit. So this would be the entire conductivity itself as well. And it is infinite, as befits a superconductor. Of course we have no impurities present in our calculation either, so we could expect this to be the case anyway.