**Thermal Equilibrium Properties**

Want to explore those calculations in a little bit different way, where we write the system’s excitations as ones explicitly above the ground state. The Hamiltonian we’ve written down comes from the Particle-Hole file in the Excitations folder. This calculation will be useful to compare to when we get to Superconductors.

**Appendix 1. Calculating Thermal Properties using the Particle-Hole Model**

So we found we can write the Hamiltonian for a formally free system as constituting excitations above an NF-particle fermionic ground state (instead of above a 0-particle ground state/vacuum like we do above). We write it as:



(EGS = 3NEF/5 basically) So particle spectrum is (εF,∞), and hole spectrum from (-εF,-0). Ok then how would we get L for this guy? It complicates things because the number of particles in the system doesn’t equate to the occupation numbers of the ‘excitations’. So let’s work it out? So our ‘base’ state would be the NF-particle fermion ground state. And the occupation numbers for k<kF would actually decrement particles, whereas those for k>kF would increment particles. Given set of occupation numbers, qk, the energy of the state would be:



Also, number of particles would be:



Therefore the grand canonical partition function can be written (presuming fermions, again),



Then doing the sum, we have:



And then we can then form the Landau free energy of the system,



Can form the expectation of the number of particles:



And so we can say for the hole and particle (excitation) occupation numbers.



So particles and holes have different occupation number formulas. But will note that,



which makes sense. And,



where NF is number of particles up to the Fermi Level. Internal energy would be, therefore:



which is consistent with:



just to be sure. And entropy,



So,



and so,



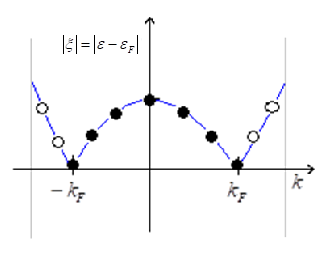
Nifty formula. Note how it’s basically S = -kΣplnp, where p is probability of probability of every state it could occupy. Or something. This form of S is conserved across all ensembles.

**Appendix 2. Calculating Thermal Properties using the Particle-Hole Model**

Let’s do this again, for the other H I wrote that describes electrons in a crystal, ‘cause it’s of general SM interest, and we’ll be a useful point of comparison when get to superconductors. Here we take the formally free particle H, but set the 0-energy mark at the Fermi surface. So H takes the form…



(EGS = (3/5)NFEF – NFEF = (-2/5)NFEF I believe) Energy spectrum looks like this:



So then how would we get L for this guy? It complicates things because the number of particles in the system doesn’t equate to the occupation numbers of the ‘excitations’. So let’s work it out? So our ‘base’ state would be the NF-particle fermion ground state. And the occupation numbers for k<kF would actually decrement particles, whereas those for k>kF would increment particles. Given set of occupation numbers, qk, the energy of the state would be:



Also, number of particles would be:



Therefore the grand canonical partition function can be written (presuming fermions, again, so qmax = 1),



Then doing the sum, we have:



And then we can then form the Landau free energy of the system,



Can form the expectation of the number of particles:



And so we can say for the hole and particle (excitation) occupation numbers.



We’ll have again,



and,



So particles and holes have different occupation number formulas. Internal energy would be, therefore:



And entropy. Can see we’ll get:



Can introduce a density of states, as was done in earlier file. We found (where that ξ below is presumed positive – didn’t feel like writing | |),



But…we can’t really use it like we’d want, because the occupation number formulas np and nh are different. We get one formula, np, for the right branch of excitations, and the other formula, nh, for the left branch of excitations. We can put, say, the left branch in terms of the right branch, but that will sort of defeat the purpose, as we’ll get the formula based on excitations above the vacuum back again.



In any event, we can see that all that will happen with the -εF term is an extra term in H given by -εFN. And this will have no bearing on measurables like the heat capacity, pressure, etc.