**Magnetic Susceptibility**

Now let’s tackle the time-dependent magnetic susceptibility. We’ll copy the Lindhardt approach to the electric susceptibility. We’ll see that all the calculations carry over – just have a little spin part to keep track of. So let’s go back to that formula,



where A = Σiai is a many body operator representing some physical quantity which is the sum of single particle operators, *a*. And |ψn(t)> are the time-dependent single particle wavefunctions. So then we can say for the electron charge density,



We shall find that <rσ´´|ψkσ(t)> ~ δσσ´´, so taking advantage of this fact, we can simplify this to:



Now let’s look at the first order correction to the wavefunction. We’ll turn the perturbation on adiabatically, so we’ll have something like V(r,t) = -gγ**S**·**B**(r,t)eηt = -gγSzBz(r,t)eηt, letting **B** point in the z direction. But then note we’ll also be adding a blue exponential factor, consistent with how we treat the time-developed quantum distribution function (see Stat Mech/Kubo Formula),



(and note the S in U is the time-development S-matrix; it’s not the spin matrix) This is necessary so that we eliminate the ephemeral effects of turning the perturbation on suddenly, and isolate the steady state response. We’ll also use the phase convention associated with infinitely timed potentials (see Quantum Mechanics/Time-Dependent/Interaction Picture file), whereby the system’s time evolution in the absence of a potential reduces to what it would look like if evolved at time t = 0. Finally, our potential includes contributions from charges in the system. So it is the total potential. Need to work out what the first order correction looks like,



where Szσ = (1/2) for spin up, and (-1/2) for spin down. Continuing,



And so,



As advertized, <rσ´´|ψkσ(t)> ~ δσσ´´. So filling this into the magnetization, we get:



Recall:



So the zeroth order term will give us zero, because the Trace of **S** is zero. So out to first order term, m is given by:



where c.c. stands for complex conjugate of the preceding term. Going to change k´´ → k´.



Filling in the wavefunction,



Can extend time integration to +∞, by writing,



And I’ll fill in the c.c. term now,



Now this takes the form of a convolution, and we can say, taking the Fourier transform of both sides (maybe see EM folder/Metal TD for an example?),



These separate FT’s are:



So filling these in,



Let’s change variables in second one to k → k + q. Then, we have:



Well, looking at **S** again,



we see that **S**σσ = 0 + 0 + Szσ. So we have:



where s = ½. Now recalling the definition we made in the Electric Susceptibility Properties file,



we evidently have:



at least up to this level of approximation. Like in the Thermal Equilibrium folder, where we calculated the magnetic susceptibility, we’ll recognize that this isn’t properly the magnetic susceptibility, because it’s not what’s proportional to **H**. I’m not sure it’s kosher to make the same manipulations we did in that file, when we could take B to be uniform in the region of interest, but….anyway we’ll say:



So the susceptibility would really be:



So, going to faux-Gaussian units whereby μ0 → 4π, and taking g = 2, s = ½, and γ = μB (in our units where ℏ = 1) we have:



We can take the q → 0, ω → 0 limit and recover our parametric spin susceptibility in the Thermal Equilibrium properties file. We have:



And recall from the Electric Susceptibility Properties file that:



And so we should have:



as we do (in appropriate units). On the other hand, borrowing from the TF file, in the small ω, q, but non-zero, limit, we have:



What is this in MKS? I think,



while in the large ω, small q limit, we have, from the Lindhardt file:

