**Excitations**

Ex post facto, it’d probably be easier to get the GF’s following the procedure in the Quantum Fluids/He3 superfluids file, where we simplify the Hamiltonian using the MFA first, and then get the GF’s. But I guess it’s nice to see it done both ways.

**Mean Field Theory**

We’ll use the BCS model:



Again, the focus of our efforts will be to attempt to determine the single particle Green’s function.



And we will use MFT to do it. It is interesting he says that mean field theory can describe SC so well. He says that it usually does a good job when the range of the interaction is large. He says that here the interaction is not so large but the radius of the ‘bound states’ of the pairs *is* so it works rather well. Our options are to apply MFT directly to H, or to the GF PDE instead. The easiest option is the latter? We will need to examine three different GF’s to get self-consistent equations. These are:



These are the only unique GF’s (though G↑↑­ = G↓↓). Note G’s operators have the same momenta, and F’s opposite momenta, which follows from Translational symmetry. It is alleged that F, F† are even in their k argument (this should follow from Parity symmetry). Some more analysis of the consequences of symmetry on GF’s is done in a later file. To get the equations of motion we need to figure out the equation of motion of the operators. So, leaving off the θ factors (in V) for convenience, putting in a chemical potential, and going to imaginary time, we can update the equation of motion formula in the 2nd quantization file. So let’s work out the time-development of the operators,



and of course,



So gotta evaluate,



Let’s just do the first part,



and now the potential,



So altogether,



If we dagger both sides of this, then we have:



and so,



So then,



Now we’re in position to get the GF’s. I start with G and I think I’ll do the two separate spin cases separately. I’ll start with σ = ↑.



the two δ’s combine to give the anti-commutator of cc† which gives 1 (can set τ´ = τ, cause of δ, and then anti-commutator is preserved by time-development). Then,



and finally,



Now we can do a mean field approximation by using Wick’s theorem on the fully time-developed operators. Of course this isn’t strictly valid. We’ll include contractions of cc and c†c† as the states against which we’re taking expectations are superpositions of many-body states and so this can happen.



Now in a translationally invariant state, we should have momentum conservation in these expectations. Should have spin conservation too. These claims are validated in the Symmetry Considerations file. So we can say,



Now the blue guy (n doesn’t actually depend on time) will just renormalize the chemical potential, so we’ll absorb it into it: ξk → ξ´k = εk - μ´ I guess. Now define,



and we have:



I’m going to presume we get the same thing for down spin, like we did in next file. Actually, I’m just going to presume we get the same results we did for all the other GF’s in the next file, ‘cause I don’t want to do all that work again.





Or, setting τ´ = 0, we have:



These 3 equations are known as Gorkov’s equations. Now let’s take the Fourier transform, ∫0βdτ eiω\_n·τ, of our three equations. For instance, the first would come to:



Doing the middle one, we get:



where we used the periodicity property of the fermionic Green’s function in the penultimate line. We can do the last one similarly, and we get, all total:



The first and last can be solved.



We find:



and F can be determined from these. So altogether:



Let’s just make sure that the middle equation we didn’t use is consistent with these solutions. So from the middle equation:



Good. Now Δ is determined by the self-consistency requirement (the ‘gap equation’). Recall we defined:



In terms of the temporal Fourier transformed FC\* highlighted above, this is:



We can do the frequency sum using the Matsubara thing,



So,



Since:



We can say,



Now filling in the potential,



So now we postulate,



Doing so, we get:



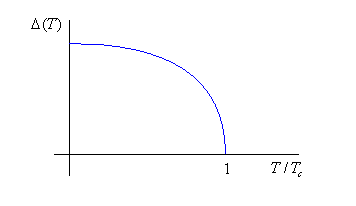
Converting to integral (Δ is constant within this range),



This is the same equation as we had for the gap when we studied the excitations using MFT (refer to MFT file for solution). We find,



which looks like this:



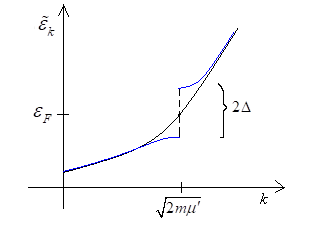
The gap quite obviously functions as our order parameter. I imagine Δ(T)/Δ represents the ratio of electrons which do form Cooper pairs to those which are available to form Cooper pairs (and these are all electrons within that μ ± ωD window?)? This interpretation would make some sense since only electrons close to the Fermi surface participate in the current anyway, and I believe that of these electrons only a fraction of them superconduct – the fraction going from 0 at Tc to 1 at T = 0). Based on the pole positions of the GF’s, we see that the quasiparticles have dispersion,



which means that a gap of width 2|Δ| (about the Fermi surface) has opened up in the spectrum, as discussed in previous file, since:



displayed below (μ = εF):



I suppose it is this gap that prevents the electrons from being scattered? The density of states is determined from the spectral function. We have:



We can turn this into the density of states (w/r to position) by forming the operator A and projecting onto the position basis:



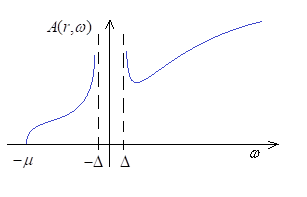
Don’t really want to work this out. Okay fine,



(θ is Heaviside function) Close to μ we can say,



So we get something that looks like single particle d.o.s. with sqrt singularity at the gap (and remember ω = basically and so would be zero at the chemical potential)



In fact F(r) takes on a complex value even though nothing in the Hamiltonian specifies what the phases should be. This is an example of spontaneously broken symmetry (akin to the breaking of rotational symmetry by the magnetization in a ferromagnet).