**Single Particle Excitations**

So we’ll examine the single particle GF in position space,



within context of Hamiltonian (δ ≠ 0),



**Symmetry considerations**

Before we begin, I’d like to verify some things, namely that the GF is should be expected to be diagonal in spin indices. Can check out the Quantum Mechanics/Many Particles/Identical Particles/2nd quantization in position/momentum basis files for some details on how these ‘symmetry’ operators act on creation/annihilation operators.

**Consequence of Spin Rotation Symmetry**

Now let’s consider a 180o rotation of spins (just translating our results for continuous position space operators to discrete position space operators)



Checking if H is invariant w/r to this operation, can’t see why it wouldn’t be:



and so we have:



Therefore H does have this symmetry. And as for the GF, this implies,



So we have: G↑↑ = G↓↓, and G↑↓ = -G↓↑. Now consider π rotations about the x-axis. We have,



and so H would go to:



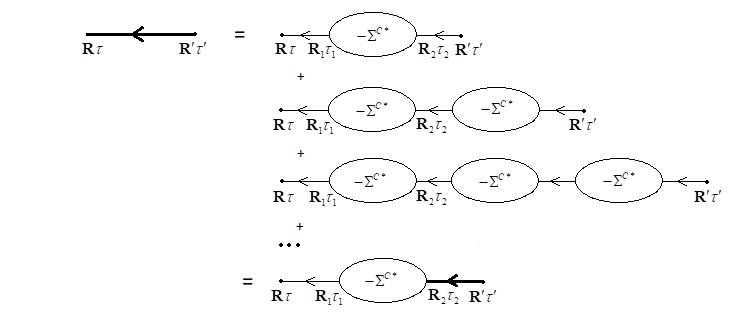
So it checks out too. And consequence would be:



So we have: G↑↑ = G↓↓, and G↑↓ = G↓↑. Comparing to the consequences of the last rotation, this means we must have G↑↓ = G↓↑ = 0.

**General Self-Energy Structure**

Now we’ll look at how the interaction ‘renormalizes’ the single particle excitation spectrum. This comes through the self-energy. The complex time GF would be:



In real space, we’d have quite generally (implicit matrix multiplication if putting G’s in spin space, but from our analysis above, we don’t need to as there is only one non-zero GF):



and since there is no single particle potential, the GF’s will be a function only of the difference of their arguments. Then so will the self-energy. And so then the spatio-temporal Fourier transform will give us:



We can solve this equation to get:



which, upon insertion of our familiar G0C\* result, gives us:

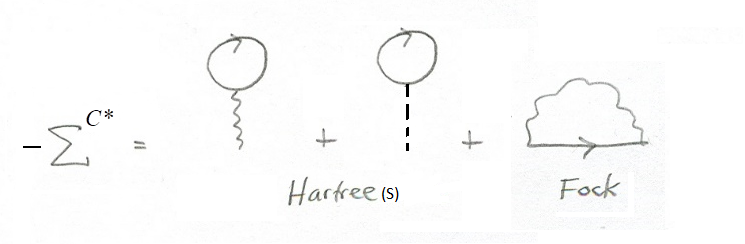


where ξk = εk – μ. By analytic continuation, the retarded GF is given by:



**Self-Energy Quasi-Perturbative Expansion**

The first order approximation to the self energy of a system of particles interacting via a two body potential is the Hartree Fock approximation represented diagramatically below:



Unlike in the nearly free model, the background, as it’s not approximated by jelly, doesn’t cancel out all V(q = 0) contributions. So we do indeed have Hartree terms in this model. Also, we only have one Fock term (the wavey one, but not the dotted one). Since the V(onsite) interaction has different spins on its two vertices, we cannot connect them – or at least we get zero when we do, ‘cause GC\*(0)σσ´ = 0 for σ ≠ σ´.

**The ΣH term [first order term]**

The first term is called the Hartree or ‘tadpole’ diagram. Since a closed fermion loop represents (after summation over spin indices) a factor of the total number density, this term represents the direct interaction between a particle at x1 and another at x2, averaged over all x2 – in other words, the average interaction of the particles with the one at x1. Applying the Feynman rules to this diagram results in (note we don’t get a factor of two when summing over spins for the onsite term because it’s restricted to be only one spin – the one opposite to the bottom vertex):



where N0 is the total number of particles, and 0 = N0/N is the total number per lattice site. Now ΣnF(εk) is really Σk<ckσ´†ckσ´> = Nσ´, which is the number of the electrons occupying the spin state opposite to that on the bottom vertex (presumably σ) since the onsite interaction requires the vertices to be of opposite spin. Presumably this would just be half the total number of electrons, which is however many we decided to put in the lattice (min 0, max 2N). So I guess we’d have:



**The ΣF term [first order term]**

In analogy with the nearly free electron analysis, this term should be,

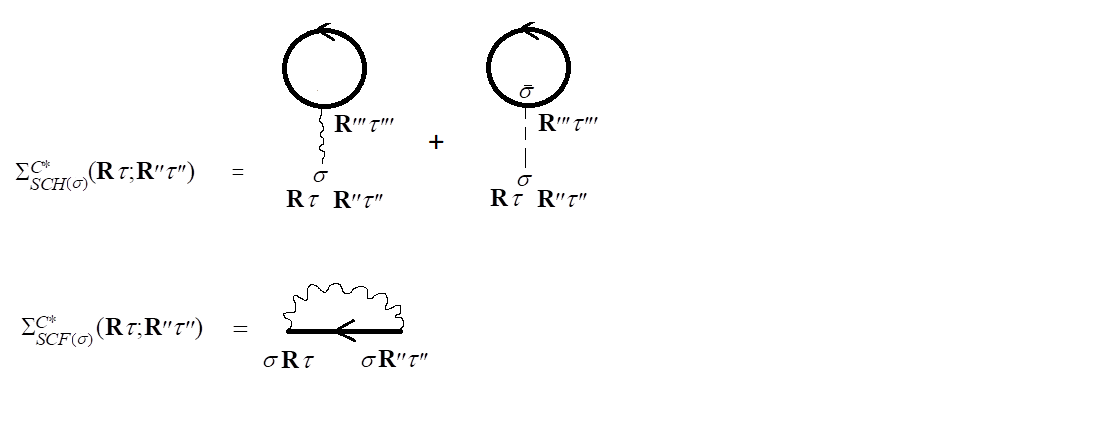


**Self-Consistent Hartree-Fock approximation**

To improve our approximation, we could go for a self-consistent HF approach. The self-energy equation is:



And for the self-energy we use:



In energy/momentum space, this would be, generalizing from our result above:



where nσ = <ckσ†ckσ> = (1/β)ΣωnGσ(k,iωn), and we’re writing it as a spin-dependent self-energy (though should get the same result regardless of which way the spin is pointing), to observe the technicality that the first n in Σ involves the particle occupation numbers for the *opposite* spin, , since we’re dealing with the onsite interaction term, which disallows same spin vertices. It is possible that n↑(εk) is different than n↓(εk) if the energy spectrum has a dip, say, like we did find in the free-electron case in the HF approximation, which makes it energetically favorable to fill spins all down or all up first before going back and filling in the opposite spin. Still, without an external field, or some internal stochastic fluctuation, there is nothing to prefer filling one spin orientation first vs. filling the other first, per se´. Then our equation comes to:



We could, say, sum over iωn to obtain a self-consistent equation for the average spin occupation number, using the fact that ΣSCHF doesn’t depend on ωn…



And so,



Anyway…we’ll use this in later file…