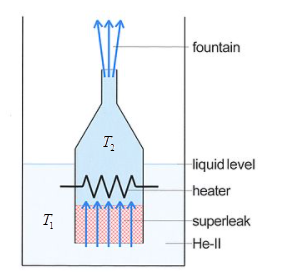
**Thermal Equilibrium Properties**

Gonna discuss just a few properties peculiar to liquid 4He. I’m sometimes using term superfuid to describe just the condensate, and sometimes I think to describe the condensate+normal fluid, i.e., He II in general.

**Fountain Effect**

Suppose we have a container of liquid Helium (in He II phase) as illustrated below. Say the temperature of the fluid within is T1. And then say we place this nozzle inside, and heat it to temperature T2 so that there is a temperature disparity between the liquid He within the nozzle and that outside of it. Let’s say the nozzle is also fitted with a coarse ‘super-leak’ through which the condensate can flow (because it has no viscosity), but through which the normal part cannot. What is the consequence of this?



So between the two compartments, the balance equations are:



And filling this into the composite entropy:



If there were heat transfer, then we must have:



But in our case there is no heat transfer; the condensate part is the only part that can pass through, and it cannot transfer entropy (because it has none), and so it cannot transfer heat (maybe see Stat Mech folder/j\_q options). Or we could make the argument by defining the condensate’s energy (the zero-point energy) as zero, in which case since it isn’t transfering energy by definition, and so it’s not transfering heat. I do think we’re just considering a small-ish time scale here; surely heat could transfer from normal component to normal component eventually via conduction, but heat transfer via convection is typically faster and we don’t have that here because of reasons we’re elucidating. So practically, chemical potentials equalize but temperatures don’t. So we just have:



(makes one wonder how entropy is being maximized, which makes μ’s equalize, and yet entropy is not being transferred?) Can write μ as G, whose differential relationship is (see Thermodynamics Folder/Potentials)



where the underscore means *per particle*. This is a little sketchier (maybe can say that there is a smooth-ish transition in intensive properties across the interface sufficient for G to possess a differential), but the chemical potentials, and hence Gibbs free energies are equal, so then dG across the interface is zero, and so,



where s is the entropy density. And so can say,



So there is a pressure differential across the interface. And it’s higher where T is higher. Parenthetically, let’s note, however, that if we allowed heat transfer, then we’d have T1 = T2 → p1 = p2. So it is this unique circumstance that we can have particle transfer w/o heat or entropy transfer which generates this pressure difference. Another comment: I’m familiar with pressure going up because temperature goes up, as this is what happens with gasses. But a liquid’s pressure doesn’t depend much on temperature. So it seems, rather, that the pressure is going up as temperature goes up, because density goes up. That’s a little counter-intuitive to me. But this would mean that the higher temperature region is sucking superfluid into itself, from the lower temperature region, which I guess it is. This pressure gradient, coming from the temperature gradient is significant, and can exert a force large enough to support a column of liquid He 56m high (it helps that it’s 1/10 the density of water too) against the downward force of gravity. So if the gradient is large enough, certainly we can get that fountain effect illustrated above.

**Helium spilling over top**

I haven’t seen anyone say this, but this would seem to explain how liquid He can escape from its container, if it’s open at the top. So there is microscopic wetting layer due to evaporation on sides of the container (I stole this picture from Wikipedia too):

A picture containing text, wall, indoor, black

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This wetting layer is very thin, but does provide a theoretical path for fluid to flow. But the normal part of the fluid, in the ladle, would find the viscosity too high. Not so for the superfluid. And so this situation is like the coarse plug in the previous example. Then, I imagine there would be a temperature gradient from the bottom of the ladle to the top, say, as the bottom is likely at whatever T < Tc, with the temperature getting progressively warmer as we go to the top of the ladle. This would provide the temperature gradient induced pressure gradient that the superfluid needs to be ‘sucked’ towards the top.

**Thermomechanical effect**

Another interesting phenomenon is the thermomechanical effect. Basically, if we place a fan in a closed container, and heat the left end of the container, the increase in temperature will draw superfluid towards itself, i.e., to the left. Since there can be no net mass transport, that means normal fluid must flow to the right. So we have superfluid flowing to the right across the propeller, and normal fluid flowing to the right across it. These rates are equal, but only the normal fluid is viscous, and so only it will turn the propeller when it flows. Picture of this below, stolen from (Steven Simon – Oxford University 2019)

Text

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**Heat Transport**

Usually heat transport and such is a non-equilibrium phenomenon, but in this case we can get heat transport in the equilibrium setting. It’s kind of related to the previous example. Consider the temperature difference up above in the Fountain section again. It causes a pressure difference, as discussed in the previous examples, which causes superfluid to flow to the left. But this time there is no high viscosity (to the normal fluid) obstruction to prevent the normal fluid to flow. And it will flow in the opposite direction, to the right, carrying this extra heat, to balance out the temperatures, and to try to bring the entire fluid to thermodynamic equilibrium. So to recapitulate, basically superfluid/condensate flows left to balance out the chemical potentials, and normal fluid flows right to balance out the temperatures. So we will get net heat transfer to the right.

If we just momentarily heated up the left region, like with a delta function pulse or something T(t) = (T+ΔT)δ(t-t0), then this would generate a heat pulse that would be carried away to the right by the normal fluid just like a momentary density spike on the left ρ(t) = (ρ+Δρ)δ(t-t0) would carried away (by both fluids). In other words, it would be carried away as a sort of sound wave. But unlike with the density spike, where the sound wave would consist of density oscillations. The heat spike would create a sound wave consisting of entropy/temperature oscillations. This is called second sound. And it’s illustrated below (stolen from Steven Simon again).

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A (first) sound wave consists of mass density oscillations of both fluids, oscillating in phase with each other. At say, the beginning (i.e., far left side of first diagram), they both move to right at t = 0, and then at t = T/2 they both move to left. In contrast, in the (second) sound wave, the mass density oscillations of both fluids are 180o out of phase with each other. So at beginning (i.e., far left side of second diagram) at t = 0, normal moves right while super moves left, and at t = T/2, normal moves left while super moves right. So there is no density wave. But there is an entropy wave. At beginning at t = 0, normal moves right carrying heat, and at t = T/2, normal moves left carrying heat.

Because of this second sound property, superfluid He has very high thermal conductivity and is frequently used as a very effective cooling agent.