**Tartar**

He says that Mott, and Twose predicted electron localization in 1D via random potential, even if arbitrarily weak. The basic reason is that forward and backward scattered waves remain coherent (at T = 0 anyway) and then form standing waves of finite length, and presumably exponential tails, and so only an exponentially small current can escape. This localization length (i.e. the size of the wavefunction) is around the mean free path, so ξ = ℓ. Thoules predicted it would do so if even if the wire had finite thickness. In this case, the number of channels increases to N, instead of 1. The wider area means more room to maneuver, less chance of overlap and therefore a larger localization length. Nonetheless, the states will be localized, which will be apparent as L > ξ. For L < ξ however, one would see g given by the Drude formula, with weak localization corrections to that, etc. This means that if take L small limit, and then g small limit, you will get the Drude and then weak localization corrections. Such corrections can be calculated via diagrammatic perturbation theory (GF) and one obtains a temperature dependent reduction in σ which reduces to e2/h or so when T = 0. One also obtains a magnetic field dependent enhancement (since this tends to reduce phase coherence). To move beyond this to the large L limit, we have to move beyond perturbation theory which breaks down in this limit. Why? (Well corrections blow up and eventuall make <σ> < 0 if let it). So at this point we turn to a more macroscopic approach, like DMPK equation. But here it is assumed that the transmission probabilities are uniformly distributed (isotropy assumption) which isn’t necessarily true. He says that <g> and ξ have been calculated in 3D? (or any dimension?) using the nonlinear σ model.

However, interest has shifted to P(g) because the one-parameter scaling theory doesn’t seem to be compatible with <g> or g\* since these quantities are not representative of macroscopically identical samples. In fact in the insulating regime, the fluctuations grow with length L, rather than decrease, whereas in the conducting regime they are fixed. In either event, they don’t self-average and so it would be nice to construct a single parameter scaling theory based off of P(g)’s scaling, rather than ‘g’. He says that in 3D conducting case (using the nonlinear σ model), the conductance distribution has been shown to be Gaussian, with log-normal tails. And in 1D the distribution has been shown to be log-normal at all lengths, while in Q1D, it is log-normal for lengths > ξ, for weak disorder at least. Perhaps this is why it was suspected that the conductance distribution would be log-normal in the strongly insulating case. He says that it is reasonable to expect this to be the case b/c the sample is basically cut into pieces of size ξ by localization. The conductance through the sample would then basically be the product of the conductivities of each, so: g = g1g2…gn. Therefore lng would be an additive random variable, and since we’d expect them to be independently distributed, we’d expect to have a log-normal distribution (though this turns out to not be the case in 3D). He gives a reference (48) that claims the one-parameter scaling breaks down near the critical point of 2D magnetic field samples. And so even a one-parameter scaling of P(g) seems to be questionable. Huh. He also says that even using perturbative (GF) techniques to study WL corrections in 2D, 3D have a problem in that they don’t automatically conserve current (I guess they do only perturbatively?). And so it remains an advantage of the macroscopic scaling approach that it can a priori incorporate this symmetry into the analysis.

He says that even in 3D, the behavior responsible for localization is coherent backscattering (in momentum space) and this is a 1D phenomenon (k → -k). And so it seems reasonable that we ought to be able to apply 1D techniques to 3D to work this out.

**Prelude**

So check out the Q1D scattering file in QM for his technique on deriving the SDE for t, r, tʹ, rʹ, and M, S generally.

**Interlude**

So we have evolution equations for the M’s and they are linear. Why not use these alone? He says even if we have solution, they are hard to use, cause how do we extract g? Seems like there are possibilities…

But if difficult, then we could, like we did, construct equations for t, r, tʹ, rʹ. These are closed as well, and more directly relevant to g. But these are not linear, and so perhaps more difficult to solve. We could write (coupled) FP equations for them, but they are not easy to solve either.

Another possibility is to go to T = tt†, which is even more relevant, but these are not closed of themselves (and its average is definitely not closed), and perhaps not obvious what extra set of variables we should add to them to get closed equations.

Another possibility is to consider moments <ΠTmn>. These do constitute a closed set. But they are infinite set. So how do we compactify this set of equations?

**Transforming <rʹ>, <t>, and <products> to an Ito derivative**

Comparing to Mello’s formalism, this seems strange because <t> was observed to exponentially decay, but here it would appear to oscillate rather. Same with <r´>, which was determined to be 0 at all lengths I think. What would happen if we turn this equation into an Ito derivative? These are the two equations, matrix notation suppressed,



Which is,



Now we needn’t expand the dz coefficients, as corrections are higher order than dz. But the other terms…



The ½ is for the fact that dWαβ(zi) only goes from zi to zi+1/2. Keeping terms to first order in z,





Then we have to fill in the derivatives:





Simplifying,



So then,



It follows that:



The evolution of the average, <t>, does not match Mello’s result, quite. Eliminating fast terms, we’d get:



Well this is exponential decay of some sort, depends on the eigenvalues of the coefficient matrix. But he uses slightly different expectations anyway. Moving on to r´,



And so,



And likewise it follows that:



And let’s do the drʹ/dz1 guy too.



Filling in the correlations:



So,



So,



Now let’s consider the derivative, and expectation of the product,



But now we have to eliminate the fast variables, basically replacing the exp(ikαz + ikβz - ikaz – ikbz) with Cαβab. So we make the replacement:



Let’s see how this replacement eliminates the fast term in the ∂t/∂z1 equation…



So yeah, and we see how the z1 term must reside on the right of t(s) to make the cancelation. What about the ∂t/∂z2 term.



And we see that the z term must reside on the left. Now with the ∂rʹ/∂z1 and ∂rʹ/∂z2 equations, we see that we need no z1 term, and that we need two z2 terms on either side.



So that explains the form of the expression – its designed to eliminate that term. Now let’s continue with the tt\* product to see what stays and what goes….Only those terms which have same number of +- k’s will survive, I’m going to ignore the z1 factor since it’s irrelevant to the z equation.



And simplifying:



If we impose the restriction in the second line, that b must be equal to m/mʹ, then we could say:



Which can be simplified to:



Now we make the definition, and use the approximation Δ = 4, which is its most common value….



And then we’ll have:



And finally if we take the expectation of both sides, we’ll have:



Says this preserves unitarity. Don’t know what that means. Well tt† is Hermitian. And if we perform expectation of this, we’d have:



So the Hermitian conjugate of the evolution equation matches the original, when t and t† are multiplied, as necessary.

This matches his result. Apparently he keeps some negligible terms present so that a closed set of equations can be developed. If we apply the same technique to the r’s, then it seems we should get:



Now we can proceed with an equation for the transmission amplitudes:



And now, eliminating terms that should oscillate out…we would get:



So we have:



He splits up the C-factor to get:



This matches his result, except for the fact that he seems to replace Δ with 16 now, instead of 4. So not sure what’s going on here. And if we do same to |rmn|2, it seems we should get:



Which should simplify to (I’m keeping my 1/16 factor that he has jettisoned)?



**Angle resolved scattering formulas**

We’d like to get formulas for <|tmn(z)|2>, and <|rmn(z)|2> in the large z limit. We go back to d|tmn|2/dz.



We’re going to do a 1/N expansion. So we’re going to explicitly assume a (large) Q1D scenario.

OK so let’s recall T = Σab|tab|2. In ballistic limit the total transmission of each channel, Ta, is 1. So |tab|2 ~ 1/N. In metallic regime, this decreases slightly, but is still |tab|2 ~ (1/N)(ℓ/Lz), and in insulating regime it’s something like <|tab|2> ~ (1/N)e-L/ξ. Note that we have ruled out already describing transverse localization since our approximation scheme is considering the transmission amplitudes to all be more or less the same, whereas transverse localization would imply a bunch of 0’s and a few 1’s. Can the approach be modified to account for this?

Reflection coefficients are like this: Ra = 1 – Ta. So in ballistic limit, Ra ~ 0 (perhaps O(1/N)?). In metallic limit, Ra = 1 – Ta ~ 1 – ℓ/Lz → |rab|2 ~ (1/N)(1-ℓ/Lz). And in insulating regime |rab|2 ~ (1/N)(1-e-L/ξ).

So I guess first term is O(1/N). Second term is (1/N)·Σβ 1/kβ·(1/N). Now Σβ 1/kβ is order N (see Appendix – but even still, if considered 1/k = 1, you’d still get N). So entire term is O(1/N). The third term is (1/N)·Σαβ 1/kαkβ · (1/N)(1/N). The sum is order N2 now, and so the whole thing is order 1/N as well. Now the fourth term? I will presume that the only terms that significantly contribute ones where we get moduli of r’s and t’s, like with Maslov’s UCF thing. Then we would have a δαβ term again, basically, which would make the whole thing: (1/N)Σβ (1/kβ)(1/N)(1/N) ~ 1/N2. So this is a lower order term. And last, we’ll note that Tr(τ) ~ N, so the first two terms are O(Tr(τ)/N2), whereas the third is O(Tr(τ)/N3) about. This probably isn’t as sophisticated as his analysis, but it’ll work. Next, he means to estimate the derivative. If we differentiate |tab|2 in the metallic regime, we bring down a 1/Lz, and in the insulating regime, we bring down a 1/ξ. Now in the large Lz metallic limit, we could say Lz ~ √N, and in the insulating limit we know ξ ~ N. So it would appear that the derivative is smaller than the O(1/N) terms too, though larger than the O(1/N2) terms perhaps. Keeping the derivative for now, we have:



So we’ve got:



Where we’ve made the definitions:



And then, the function we get by making the replacement km = kFμm, is, for future reference (μ is ratio of k|| to kF, basically cosθ between them),



Current conservation provides a relationship between the two:



And switching to μ variables would give us:



Now let’s do same with <|rmn(z)|2>. I’ll expand the products:



And do some simplifying. In the last two I’m going to use rʹ = rʹT.



First term is O(1/N). Second O(1/N2). Third O(1/N). Fourth O(1/N). Fifth O(1/N). Sixth O(1/N). In the last line, the first term is O(1/N) if m = n. But the one is O(1/N2). Now derivative is of higher order by 1/Lz (metallic regime) or 1/N (insulating) and can therefore also be neglected at lowest order. So gathering these together we have:



Continuing, keeping the derivative for a while:



So then,



So now we’ve got:



**Asymptotic limit of <|t­mn(z)|2> and <|rmn(z)|2>**

OK let’s consider the zeroth order solution to these equations, where we neglect the derivative. This should be the asymptotic limit z → ∞. This would result in:



Now, he says that |tmn|2 ought to be symmetric, of course, and so that enables us to basically equate:



And so we can say:



We can fix the normalization constant to <T>, to write:



The sum over ρ can be smoothed into an integral (note ρ(k||) depends only on the magnitude of k||),



So we can write:



And for <|rmn|2> we’d have:



Except for that weird factor of 4 discrepancy. So,



Seems that we could work out <T> given current conservation constraints. Anyway, note that while this is symmetric, it isn’t of product form, unlike <|tab|2>. Now we can develop an equation for ρL(μ) by relating the <|rʹmn|2> to its relation to ρL(μ) itself. We consider the case m ≠ n:



So finally our equation is:



Apparently this equation has the following solution (has a derivation in the appendix – should look at it sometime):



So under the approximations made so far, we have the angular structure of the transmission/reflection intensities, up to one free parameter, <T>. We can work out some basic properties w/o the full solution, though. First, let’s note that, assuming Lz >> ℓ so that |tmn|2 is really small (in metallic regime its ~ ℓ/NLz, whereas |rmn|2 ~ 1/N), ρL(μ) averaged over all channels is:



And we can work out its moment, ∫μρL(μ), too. We start with the integral equation itself:



Which gives us:



Then multiply this by μ2 and integrate:



And so we get:



With this we can write:

Observe <|rʹmn(z)|2> isn’t equal to 1/2N in this limit. And neither is <|tmn(z)|2>. So isotropy never happens basically. Hmmmmm. Observe the ratio of backscattering into the same channel vs. nearby channel:



He relates that enhancement by 2 is due to time-reversal invariance, and the decrement is due to fact that single scattering event is its own TR self and must be included in any event, and so adjusts factor. Anyway, now he wants to improve our results a bit by keeping the derivative…

**Backing up on limit of <|t­mn(z)|2> and <|rmn(z)|2>**

Now we want to step back a bit and consider smaller Lz’s. Here, Lz < Nℓ, and so we should keep the derivative term. So we go back to:



(where s = z/ℓ) Now the factor of 4 issue isn’t here anymore? So setting Δ = 4, we get to:



And now recall:



We’re going to recast the equation in terms of this quantity:



Now it looks like he’s assuming <|tmn(z)|2> is symmetric in its indices. This seems plausible, and we’ve assumed it before. Does it follow from something? Anyway,



And finally,



To complete the equation, we’ll go the <|rmn(z)|2> expression – I think we’re assuming m ≠ n still. So,



Well, he uses the drʹ/dz1 equation rather. I don’t want to rederive all that stuff, so I’m going to just quote his result:



Which can be put through the same manipulations to get:



We’ll assume a solution of form:



Then the two equations yield:



Let’s clear the denominator:



And now plugging into the next equation:



So our solution is:



He employs another identity, which he claims follows from the two integral equations, though I don’t see how, to ascertain b = 2s0 = 2(0.71) roughly. So then for the transmission/reflection coefficients we get:



And,



In the large s limit I guess we can neglect the derivative? Then we have:



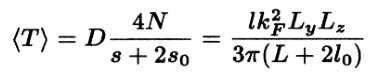
But then since we neglected the derivative, which amounts to the next order term down from the lowest, we must neglect the μm/(s+b) term as well. So then altogether we have:



This is the m ≠ n case still (at least vis a vis the reflection coefficients). Comparing to the asymptotic limit of <|tmn(z)|2>, we could extract <T>. Or, summing over all mn we get:



which is Ohm’s law. Filling in N and b explicitly we get:



Note this equation does not work at s = 0, since we should simply get N, whereas we actually get about 0.95N or so. But it does describe the transition between ballistic and Ohmic transport. He says that the b term can be interpreted as a surface resistance on the two ends, and follows from a phenomenological resistance equation:



**Fokker-Plank Equations**

Now ultimately we’d like to get a bunch of evolution equations for all the ‘moments’ of the vector variable **Y** = (t1, t2, t3, t4, …), where tn = Tr(Tn), and where T = tt†. Reason being, we will find that we can ascertain the evolution equation for the characteristic function FL(**υ**) = <exp(i**υ**·Y)>, from which we can attain moments, and also p(ξ) = <δ(ξ – **Y**)> as well. First, a few properties:



To that end consider Taa, by summing our |tmn|2 scaling equation over n.



If we sum over m as well, then we have:



And now we substitute t’s for r’s using the identity above:



Which is similar to his result, but back to the 4-factor weird thing, and also missing the diagonal C-term. What if i just analyzed the expression we got when kept the largest terms?

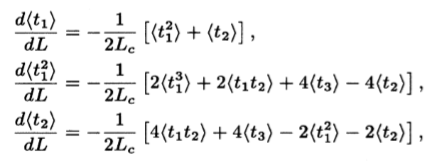


But anyway we’ll say:

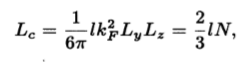


consider that sum T’s are like ρρ and sum over ρ/k → ∫ρ so maybe that’s how k’s disappear. Notice the cancelation of the T terms above – maybe that’s what he’s talking about when he mentions cancelation in text. Seems the two terms are of different orders. Right one larger (1/N) and left one (1/N2)?

For the other moments, I’ve made a bunch of attempts, but don’t get what he does, and not sure how to incorporate his previous results into the analysis, as he says he does. So anyway, he claims that he finds the following results – don’t see how…



where,



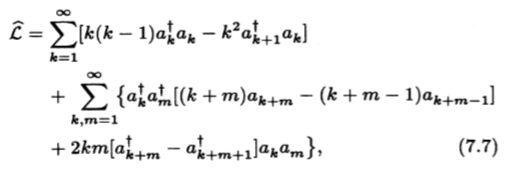
Note there is just one characteristic length here – the Q1D localization length. Why is there no 3D information in this? We can write these equations succinctly, and generally, introducing the following notation for a generalized trace, and cross traces:



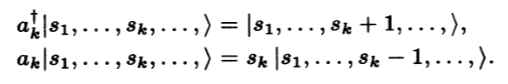
where s is a vector denoting the exponents of all the tn’s. Then we would, in principle, get a complete set of equations. General equation can be written as:



And the operator is given by:



The creation/annihilation operators do this:



We’ll anticipate a remark he makes later, that these equations are valid only a bit after L = 0, since we made the L > 0 approximation in the previous section, where we worked out the angular structure of R and T, which we needed to invoke to get these results in the first place. I suppose we would prefer a closed set of equations involving expectations, than a closed set involving Mij themselves? Perhaps it is prohibitively hard to take an average over the solution to that equation, or even to get the solution to it? Mello derived a set of self-consistent equations in terms of generalized λ-traces, where ρk = Σiλik (but k capped off at N)? And from this, determined a FP equation for the probability density of these traces. Perhaps λ is the better variable to consider, rather than T? In any event, knowledge of these traces allows us to write an evolution equation for the characteristic function:



We’ll expand it in a Taylor series,



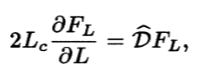
And then take the derivative.



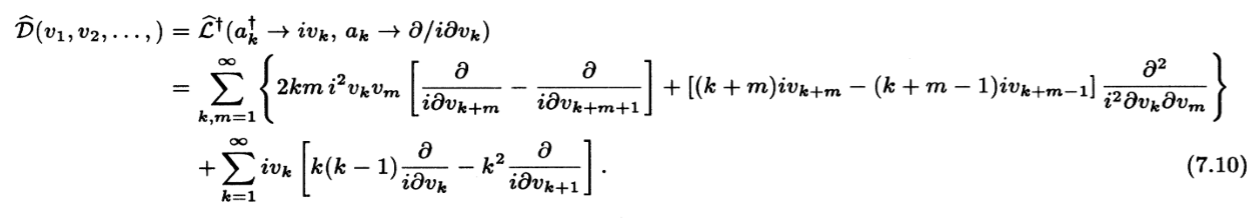
Now we have the equation for the time development of these traces. Basically they involve ak operators, and a­k† operators. The former reduce sk by one and add a factor of sk. How can this be reduced to an operation on υ?



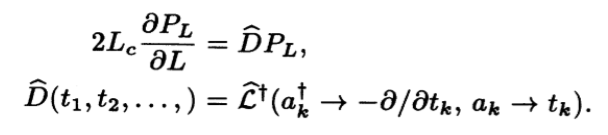
So basically ak → iυk. And ak† increases the index of tk by one. This is equivalent to multiplying through by tk, which is equivalent to differentiating w/r to (iυk). So ak† →∂/∂(iυk­). And so we eventually find:



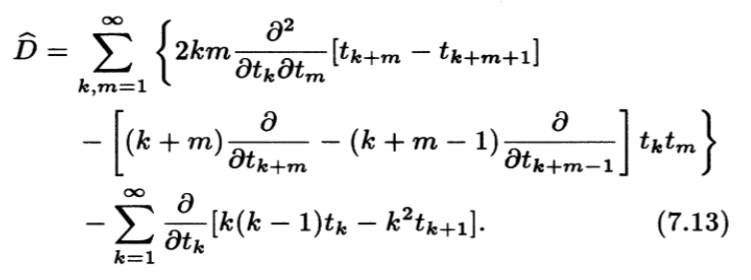
Where,



And then, taking a Fourier transform would bring us to the evolution equation for the probability density of the transmission traces PL(tk).



And, explicitly:



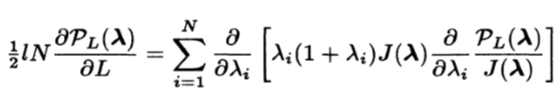
This is similar to Mello’s equation for the λ-traces. Going a little out of order, we want to demonstrate the equivalence of this approach with the standard DMPK equation. So he says, consider again tn = Tr(Tn) (T here is the matrix tt†).



The argument in the delta function is justified via:



And now he says, suppose that P(λ) satisfies the DMPK equation:



Then PL(t) so defined, will satisfy the previously encountered equation:



How would we check? Suppose we apply the ‘Dmpk(t)’ operator to PL(t)…



Seems like we’d have to do some sort of Jacobian thing to put the ∂/∂tk in terms of ∂/∂λ­k. Don’t really want to do that right now.

**Working out special cases in the various regimes**

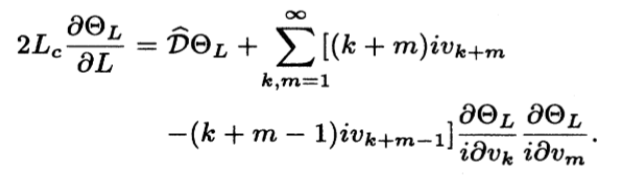
So now, to get some concrete results, without having to solve this equation, he wants to change variables to: ΘL(tj) = lnFL(υj). He makes note that cumulants can be obtained from derivatives of Θ. For instance,



And then,



Plugging this into FL(**υ**)’s equation, we get:



Is this the end of the variable transformations please? Now he remarks the conductance self-averages in the diffusive regime. This means the variance, and other higher cumulants should be basically zero. So should Θ(**υ**) basically be linear, alone, in that regime? Whatever. So we can use this equation to separate out evolution equations for the moments, tn, tn2, etc.



He develops an equation for <tn>, from the Θ equation. How so? Let’s consider it. So to get <tn> on the LHS we need to differentiate:



But to get <tn> we need to set υ = 0 too. So if we do this, then we’re left with (implicit υ = 0):



And,



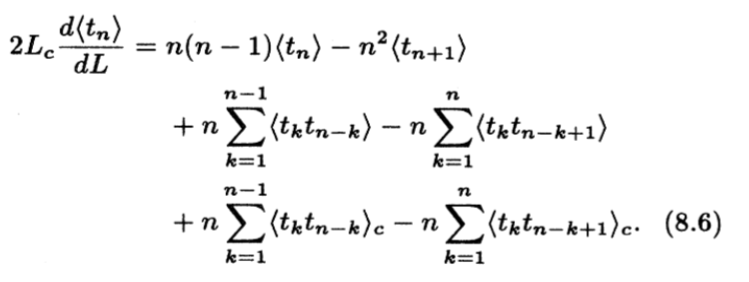
And,



And finally,



This is basically what he got, except for the separation of the t’s in the middle line.



**Metallic (Diffusive) Regime**

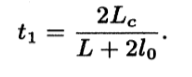
Now it looks like we’ll use a MFA, which he says should be valid in the diffusive regime, since it self averages. So,



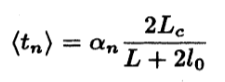
Furthermore, since all the transmission eigenvalues ought to be close to 1, the tn’s ought to be close to N. And so the first two terms ought to roughly cancel,



In light of the fact that that this equation does not apply for super small L, and so we cannot demand t1(0) = N precisely, we instead demand that it satisfies Ohm’s law for small L so that:



And with that behavior as a frame, we define:



Plugging this into our difference equation gives us:



This looks like an ‘integral’ equation, like a convolution in particular. But anyway, he uses a generating function to convert the difference equation into a differential equation. Consider:



And now….form this expression on the LHS (note we’re basically applying one version of a Z-transform to both sides),



Have to be careful when rearranging the sum.



Note that when we expand the first sum to n = 1, it naturally generates α1. And, then,



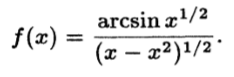
I’m kind of biasing my analysis, but just suppose I were to anti-differentiate both sides. Then I’d get:



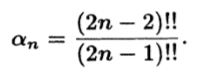
And so we have his equation:



This is a linear ODE and so can be solved, with the solution:



And we could get the coefficients by taking the appropriate version of inverse Z-transform, or by expanding in Taylor series, or by going to the penultimate line in the MathType block above, and filling in the series expansion of f(x) – we’ll get an easier recursive equation for the α’s. Either way we get:



So now we want to extract the eigenvalue density and here we use a clever trick. Note that having the average of all the trace-powers is tantamount to having the density, we just have to figure out how to parlay the information from one to the other. And we use this:



Where Tn are the eigenvalues of T. Now note:



So we can say:



Then he introduces the function:



And offers the following identity:



I’ll just plug stuff in, and hope for the best (leaving out implicit trace and expectation):



And,



In the last line did a little partial fraction thing. And then in the next line, I think I can take the small ε limit in the prefactor, already. In that case we’ll have:



And then I suppose I can ignore the z prefactors on the ε, since ε goes to zero anyway:



And so there we go. Then he explicitly calculates Q, via Taylor series. Recall tn = 2Lc/(L+2ℓ0)αn = <T>αn. So,



Anyway. So he applies the formula:



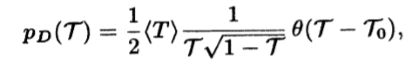
and obtains,



So,



Which matches his result, except for the little cutoff he puts in there at T0 to keep it from blowing up. I suspect it fails at that point T = 0 because our partial fraction expansion thing failed at T = 0 too.



Once we have the density, then we can calculate the expectation of any linear statistic, A(T):

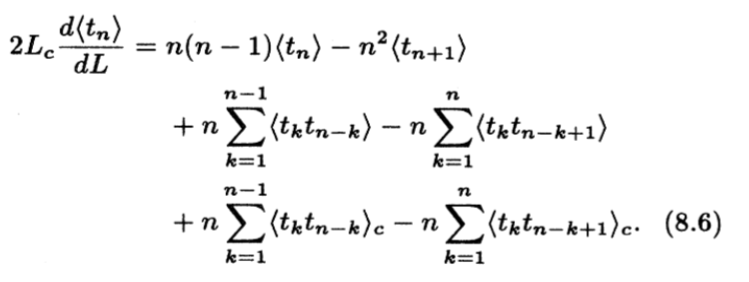


via:



**Metallic (Weak Localization) Regime**

Now we go back to the equations:



And we assume a perturbative expansion in inverse powers of the conductance. The cumulants start at 0 because in the metallic regime, the electrons should behave diffusively, meaning the correlations are insignificant, or at least should only get smaller with smaller L.



I think. And then to get the expansion of the moment, we can use the following formula:



Then he plugs this into the equation for <t1>:



Plugging our stuff in, keeping out to 1/(L+2ℓ0)2 order.



Equating coefficients:



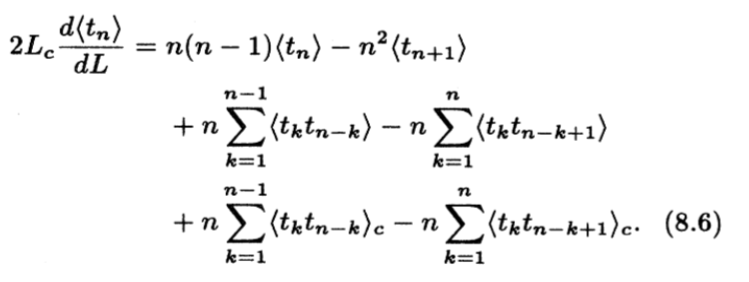
Recalling the formulas for the αn = (2n-2)!!/(2n-1)!!, we have α1 = 1, α2 = 2/3, and so β1 = -1/3, which he cites as the known first order correction to the conductance in the Q1D limit. Next he proceeds to the variance, to see if we can get the UCF. This would be <t12>c. We have to go back to the ∂Θ/∂L equation to work an equation for ∂<t12>c/∂L. And he says this is what we get:



Plugging stuff in again, but just focusing on the lowest power, 1/(L+2ℓ0)2, again,



Neato! Now if we want the WL corrections to any statistic, we will have to, in general, work out all the β’s for the <tn>’s. So he goes back to the equation:



And plugs in the expansions, probably:



So, going out to 1/(L+2ℓ0) order this time:



And,



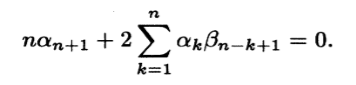
And we get the following two sets of equations.



Note how the first equation is the same as we had before. This time it emerges ‘naturally’ as the lowest order term in the equation, which justifies ex post facto our previous neglect of those first two terms. The last equation simplifies to:



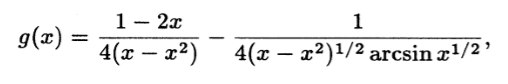
But he has a (+) sign on the last term, whereas I have a (-). Hmmm… Anyway, he reduces it to:



Remember we know what the α’s are. So we just have to solve for β’s. He introduces another generating function – again basically a Z-transform.



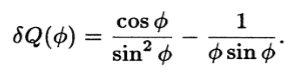
And solves the equation to get:



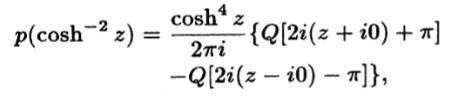
From this we may deduce the β’s. And then recalculate Q:



And he gets:



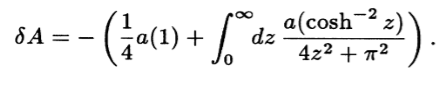
So we can calculate the WL corrections to the density using the aforementioned:



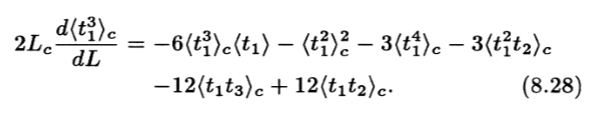
Now that we have corrections to the density, we could go back and work out corrections to any linear statistic:



And he obtains the correction:



Like before we can investigate higher order correlations. If we go back to the ∂Θ/∂L equation, then we’ll find, for instance:



We can work out the leading order behavior by filling in the usual expansions.



The lowest order term on the left would be constant at most. On the right, the lowest order is 2Lc/(L+2ℓ0). The fact that this must match the LHS means that <t13>c must be of order [2Lc/(L+2ℓ0)]-1. He claims that in general,



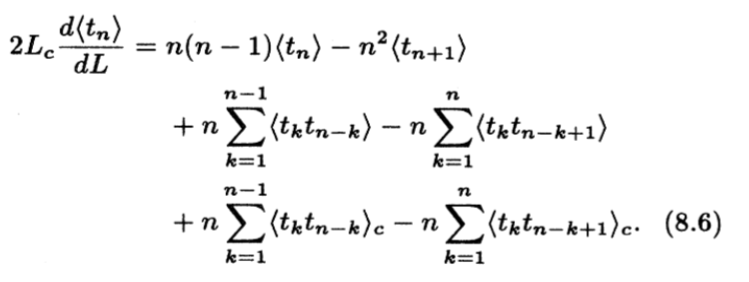
And that even more in general:



Then he proffers that this proves the conductance distribution in the metallic regime is Gaussian, since as L shrinks, all the cumulants higher than the second vanish.

**Insulating Regime**

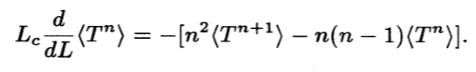
Once again, we go back to the equations:



And note that in the insulating regime the transmission coefficients are exponentially small, and so can no longer be expanded in a power series. So we need a new ansatz. Note the transmission eigenvalues Tn are widely separated, and the first one will dominate the others (in transparency). So <t1> ~ T1, and <t2> ~ T12, and <t3> ~ T13. So we may propose <tn> ~ <t1>n. So we can make the following approximations:



To arrive at his equation:



Diverging somewhat from his derivation, it seems we could work out the probability distribution function P(T), from knowledge of these moment equations. Let’s construct the characteristic function:



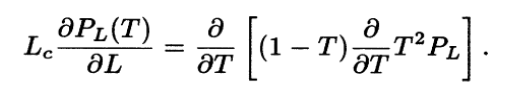
And then,



Doing Fourier transform,



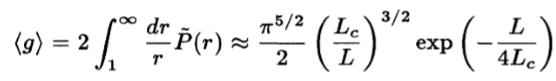
As compared to:



Which, expanded, is:



So that matches 😊. He says that this matches other people’s results. Furthermore it matches the result for a 1D chain conductance distribution, except for replacement of ℓ with Lc. Solving this equation and calculating <g> we get:



He makes comment that the most probable conductance is g\* ~ exp(-L/Lc), whereas the average conductance is <g> ~ exp(-L/4Lc). The average is much higher because of very rare impurity configurations which confer near transparency to the sample. The difference between the two highlights the large conductance fluctuations.

**Here are a bunch of (futile) attempts to generate the higher order trace evolution equations.**

First some trace properties:



**Another way to get T**

Can do this via direct matrix multiplication too?



So,



Now want to Ito-ize it:



Keeping the appropriate terms,



And,



And…



In component form,



And,



Gonna use Δ = whatever:



And,



Or,



As a check, let’s calculate <Tmm>. Have to factor out fast variables:



And,



And,



And then if we sum over m, we’d have:



Continuing on,



And this matches our previous result, sans the last guy.

**And another way to get T**

And we could also use Trace feature up front, as well as introduce the derivative conversion from Stratonovich to Ito, later. Now let’s go back and do derivative of product in Stratonovich manner for now.



Actually we want,



Now convert to Ito,



And,



Keeping the slow terms,



And then flipping the rʹ† around to the front and using tt† identity:



Distributing and combining,



Now components, and take average, and do slow terms:



And,



And the top line cancels,



And,



So final result is:



**Trying to calculate T2**

What about <T2>? Well,



Set mʹ = m, nʹ = n:



Now consider the derivative of the product:



We’re ultimately going to sum over mnmʹnʹ and take out fast terms, etc. So the two terms up top ought to come out as just duplicates of each other, and moreover, be given by our previous work. So, we should have (implicit summation over all variables):



Now doing the statistical average:



I’ll set Δ = 4.



Guess its time to see which terms stick around or not:



Putting the C-terms in there….



Now trying to clean it up a bit:



The two remaining rʹ-less terms amount to an imaginary number. But there should not be one. So ignore? Guess I will,



And,



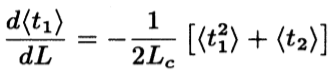
And,

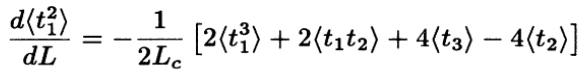


check to see if on right terms, you can use the rʹt = trT or whatever thing to get an α index on the rʹ term, to make it look like the ones on the left.



Well problem is that the t’s don’t match now – so whatever. First two terms match, more or less, but last terms i’ll just have to accept for now.





What if I just made the exponents of the rʹ’s cancel out.





**Another way to get T2?**

OK now let’s try for the [Tr(tt†)]2. So,



Now Ito-ize:



Doing a conjugaty thing on the top trace we can say:



Now going to take average and so keep only dW2 terms:



The Tr(tt†) term is already real. So slow terms will only be on ImTr[ ] multiplying it. How do you ascertain slow terms in Im()×Im()? Well,



So, keeping only slow terms…



Now doing the rʹrʹ† tracey thing in the top bracket. And use fact that dW is Hermitian,



Components,



Now group and take expectations:



Simplify a little,



And,



Splitting up the C’s, but skipping the diagonal terms?



Simplifying,



And,



Is it real? Well Tαβ = tαmtβm\*, is Hermitian.



**Last attempt at T2**

Going to use the z1 equations,



So,



And,



And so,



Now have to Ito-ize,



And,



Eliminating the term proportional to dW alone, observing cancelation of those two terms, and utilizing fact that top two terms are daggers of each other:



And noting that Tr(A) + Tr(A†) = Tr(A) + Tr(A)\* = 2Re[Tr(A)], which has no imaginary part obviously, so…and also using tʹ†= t\*.



And now using:





Which is the same as:



We’ll remember that rʹ is effectively neutral as far as z1 is concerned, and tab ~ exp(-kbz1). So the surviving terms are:



But then it seems we have a cancelation:



Finally let’s put this into components,



And,



So,





Breaking it down…keeping only slow parts…and parts that will survive averaging, keeping in mind that rʹ has no exponential parts w/r to z1 and t as -kzright index.



Splitting it up, and recalling,



We have:



And simplifying, we have:



And keeping slow terms in the product,



One last simplification:



OK now components,



And,



And,



In this case, the interesting term will be:



Maybe somehow



**Why not? T2**

So what is the general evolution equation for F(M) then?



And generalized to matrices, this would be:



So then let’s put our equation in this form,



So then,



and so,



simplifying,



Need to separate the blocks i think,



Guess could combine Σz + iΣx = Σ



Now say we want to determine the evolution of Tr(T2). Have to relate this to the blocks.



How do you take derivative of an inverse? Now from implicit differentiation:



So then,



Not eager to execute that derivative.



Yeah no.

**Can we just solve the dM/dt SDE?**

We have:



Let’s define Mʹ = exp{-ikΣzz}M. Then we’ll get:



Do Σz and Σ commute?



Nope. So best we can do is write:



This equation could be solved, in principle, if M were a single variable. It’d be:



Problem is just that we need to somehow write it as a bunch of independent single variable equations. But if V(z) isn’t Hermitian, how do we separate it? So this doesn’t seem solvable per seʹ?

Could write as a path integral, perhaps find the classical path, and the quantum corrections? Except, V(z) is not Hermitian. Don’t think that matters. But U(z) is completely general, and most methods required an input. Let’s consider the path integral approach. So,



Approximating the solution to this equation would require a bit of path integral trickery. And besides, would take us away from the FP approach.