**Excitations**

Continuing from previous previous file…recall we had:



where the gap matrix is:



Δ(2)i=± are the formal eigenvalues of the gap matrix:



and |ψi(p)> are the eigenvectors. And the gap equation self-consistently determined the behavior of these eigenvalues as a function of k (and T).



where,



Note the Δi(2)(p) are implicitly functions of the various Δσσ´(p). At the end of the last file, we said that a more convenient basis to consider was the ‘coupled’ basis, defined below:



And we said solutions to the gap equation practically decouples into separate singlet and triplet solutions. Well maybe I didn’t say that, but I am now.

**Triplet Solution**

Now we’ll look at the triplet solution. Setting the Δ00 = 0 has the following implications:



We consequently find:



And the eigenvalues are:



Yeah. And the eigenvectors of Δ(2)σσ´ are? Something we *could* work out. And then plug into our gap equation:



where,



But no. Even if we did do all this, there are apparently many(?) different solutions to these equations still. FWIW, each of the Δ1m(k) gap functions is obviously a function of k, but presumably, just a function of the direction of k, since these gap functions should only exists for k’s within the |ξk(0)| < ωH window. Since they’re just functions of (θ,φ) then, we can expand them in a series of spherical harmonics,



(somewhat confusingly, I’ve been using k to denote a vector, but now I’m going to use k to denote the magnitude of a vector, and **k** to denote the vector…mostly….and I’m putting a ~ over the spherical harmonics, because I’m actually going to use these to refer to the spherical harmonics with unit coefficient) Turns out the gap functions of the B and A phases of liquid Helium invoke only the ℓ = 1 spherical harmonics, and are said to have p-wave symmetry. So we can say,



Or written out,



The normalized Y1m spherical harmonics are given to the left, and the tilded ones to the right.

**Balian-Werthammer State**

So we’re going to make an asatz for a solution that apparently describes the B phase of liquid Helium. Our (their) ansatz is:



Now let’s see what the consequences of this ansatz are, apropos the individual Δσσ´, and Δ(2)σσ´, and the latter’s eigenvalues and eigenvectors. From before, we have:



We consequently find for Δ(2)σσ´,



And the eigenvalues are obviously:



Maybe I’ll hazard looking into the eigenvectors now? Yes I will try. These are:



So to summarize somewhat, our eigenvalues and eigenvectors are:



And now we’ll plug all of this into our gap equation. Noting that the eigenvalues, and therefore k´(i), are independent of i = ±,



Now let’s try to work this out. Filling in k, and the eigenvalue:



we have:



Now filling in the potential,



And now filling in the functional form of Δσσ´



we come to:



where can see that the angular integral will eliminate the M0 term. The angular integral over the M1 term can be done directly, since only simple trig functions are involved. But a little fancier way to do it is to recognize that:



where we use the Cartesian polar coordinate formula kx = ksinφcosφ, ky = ksinφsinφ, and kz = kcosφ. So solving for the momentum vector, we have:



So,



and so our integral is:



which is an interesting identity. We have the same identity for ℓm as well, since it’s proportional to Yℓm. It follows from this that we can say,



We can take η(k) constant over the width of the energy window,



which means it will cancel from both sides, along with the Yℓm guy, and we can also pull out the density of states, bringing us to:



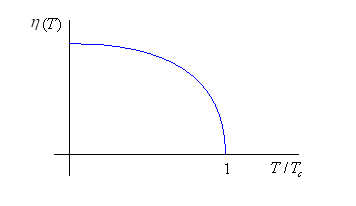
This is again the same form as the super-conducting gap equation.



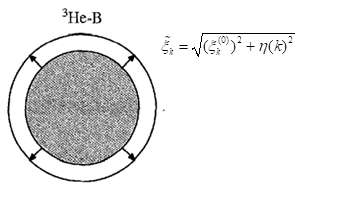
And so we can presume all of those results carry over here, just with a different effective coupling constant. We’d expect something like,



and,



Finally, we’ll note that the gap, having no angular dependence, is symmetric about the Fermi surface. So looks like this:



Now we’re going to move on to the next state.

**Anderson-Brinkman-Morrel State**

This solution describes the A phase of liquid Helium. It’s given by:



Now let’s see what the consequences of this ansatz are, apropos the individual Δσσ´, and Δ(2)σσ´, and the latter’s eigenvalues and eigenvectors. From before, we have:



We consequently find for Δ(2)σσ´,



And the eigenvalues are obviously:



Eigenvectors are obviously, once again:



So to summarize somewhat, our eigenvalues and eigenvectors are:



And now we’ll plug all of this into our gap equation.



But this time we do have two different eigenvalues. So let’s separate out our two equations:



We’ll find analogously for the other guy,



Now let’s try to work this out. Filling in k, and the eigenvalue:



we come to (letting ±1 stand for ↑↑, ↓↓):



Now filling in the potential,



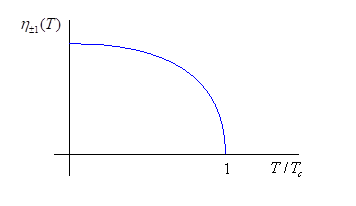
and then the functional form of Δ±1(k):



We can at least take η±1(k) to be constant within the energy window,



But can’t do the angular integral as easily this time, thanks to the sin2θk´ in the sqrt. And I’m not going to. I presume that our ansatz does satisfy the equation though. So we should again come to something like:



And we’ll also observe that this time the gap equation is not symmetric, thanks to the sin2θk dependence. Instead we get the following picture.

