**Excitations**

Going back to,



where,



and add disorder to it,



where, as we’ll recall from the Metals/Electrons/Impurities file,



Though the Hamiltonian is technically solvable exactly since it’s bilinear, the disorder average we have to take makes that unfeasible. So we’ll attempt a perturbative expansion of the single particle GF. Recall in previous files we found we could write,



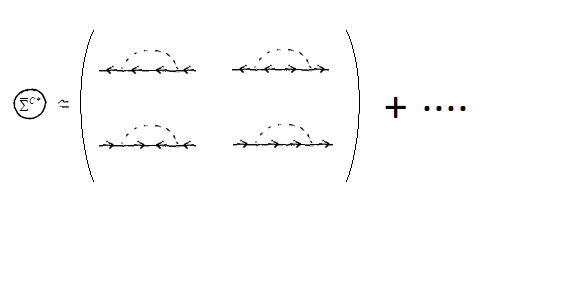
Let’s check out two approaches to getting Σ.

**Born Approximation to Self-Energy**

So in the Metals/Impurities/Excitations/Weak Disorder file we introduced various self-energy approximations. One was the Born approximation.



which in this context would be:



If we write this out mathematically, we’d get:



which is, more succinctly, recalling Pauli-matrix σz = diag(1, -1),



Let’s specialize to a delta function interaction,



Then we have:



We can change variables to q´ = k + q, and write this as:



which makes it clear that Σ doesn’t depend on k. So we just have:



Now we fill in our expression for **G**,



so,



and now we multiply top and bottom by the complex conjugate of the denominator (see similar calculation above).



So now we have:



The first and last integrals are technically doable. The middle equation contains corrections to the energies basically. This will diverge because it goes as dq q2/ξq for large q, thanks to the δ approximation to the (disorder) potential, basically. And like before, we argue that in a better approximation to the potential, this contribution would converge, but still be quite small. We’ll assume the same here. So we’ll just say:



And then the other two integrals reduce to:



Can put these in terms of spin-less density of states,



Apropos the first guy, I guess I’ll split the integral up into parts,



The density of states is approximately constant in the second integral, since ωD is so small, and I’ll accordingly approximate ε = μ´ everywhere. I’m also going to pretend that Δε is Δ within 2ωD interval of μ´ (rather than μ). That seems to be what people do….? I’ll make the same approximation on the first integral, even though it’s not warranted per se´, but then, we probably don’t care if that one’s too accurate anyway. So,



Going to do similar things for the second integral,



So altogether,



So going back to our self-energy equation, we have:



Guess we’ll write this as:



The terms proportional to Δ are pretty small, so we can neglect them and just keep,



And if we define,



Then we can say,



This compares with what we had for non-interacting disorder averaged electrons. And can write this as, to facilitate analytically continuing to the real axis,



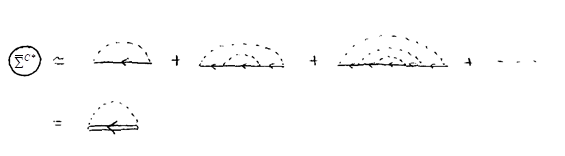
The retarded GF would be, making replacement iωn → ω + i0+:



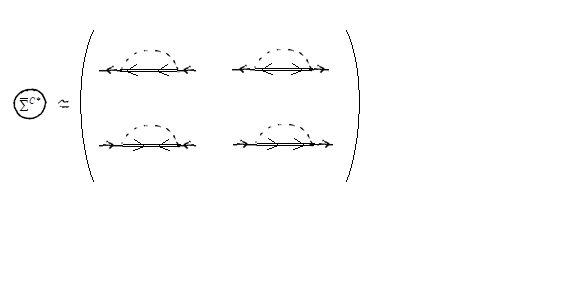
This is pretty similar to what we found for regular free electron GF’s, subject to disorder, in the Electrons/Impurities/Excitations file.

**(Self-consistent) Born Approximation to Self-Energy**

Now let’s do the self-consistent Born approximation,



In this context, we would write the self-energy equation as,



Following the same procedure as above with the first order Born approximation, if we write out the latter, mathematically, we’d get:



Again, let’s specialize to a delta function interaction,



Then we have:



We can change variables to q´ = k + q, and write this as:



which makes it clear that Σ doesn’t depend on k. So we just have:



Now we fill in our our expression for in terms of the self-energy,



and use,



so,



This is the self-consistent equation we have to solve. We can surmise a form for Σ, let’s suppose,



and then we’ll have for the integrand, which is C\*(q,iωn) of course,



where we have now defined tilde variables to incorporate the self-energy terms,



and now we multiply top and bottom by the complex conjugate of the denominator (see similar calculation above).



So now we have:



Since 1, σx, σy, σz are linearly independent matrices, we can equate corresponding coefficients,



which is, filling in what the tilde’d variables are:



The first and last integrals are technically doable. The middle equation contains corrections to the energies basically. Like in the first order Born approximation we did above, this will diverge because is goes as dq q2/ξq for large q, thanks to the δ approximation to the (disorder) potential, basically. And like before, we argue that in a better approximation to the potential, this contribution would converge, but still be quite small. We’ll assume the same here. So we’ll just say:



And then the other two integrals reduce to:



Can put these in terms of spin-less density of states,



And if we take easy way out and approximate the density of states as being constant, and given by the value where the integrand is largest (so as to capture the most important contribution), then we’d just use replace ρ(ε) → ρ(μ´). We did this for the simple first order Born approximation above.



And let’s also split the integral up, like we did above. Guess I’ll do this one at a time. So first,



Now in top one we’ll shift variables and take μ´ → ∞ as shouldn’t affect integral too much. And in bottom two guys, we’ll approximate ε as μ´,



and can do similarly for the other one,



But this makes it hard to solve for the two Σ components. A simpler approximation is to

go back to,



And just replace Δε with its value within the gap, i.e., just Δ. If we do this, and shift variables, etc., we’ll have:



and then,



To make them a little cleaner, let’s recall (in first line) and define (in second line),



In terms of these, we can write,



and,



Let’s call = n/, and take the ratio,



So,



Now we’ll plug this into one of original two equations…



Now q would then be:



And from = n/, we can see that n is equal to:



So altogether,



Interesting that the self-energy is just second order in the perturbation, even though we went to all orders via our self-consistent approximation. This seems to imply that we would’ve gotten the same result if we had just calculated the Born diagram, non-self-consistently (well we kinda did). Poles of the GFR = GFC\*(iωn → ω+i0+) determine the excitations. So let’s consider:



This is pretty nasty. I think I’ll just presume that q is close to kF, in which case we should be able to neglect Δq-Δ. Then we have:



Now let’s say c2 = ωn2 + Δ2,



Filling c back in, and changing iωn → ω + i0+,



and I guess I’ll stop here,



‘cause don’t want to bother separating into real and imaginary parts. Might note that when q is on the Fermi surface, then we have:



So disorder is renormalizing the energy spectrum, and of course turning all the former exact (MF anyway) excitations into quasi-states with a finite lifetime. But at the Fermi surface, the excitations are exact w/ infinite lifetime, and moreover, the T dependence of that gap is the same as it was w/o disorder. I think this is part of Anderson’s Theorem – that disorder doesn’t mess with the T-dependence of the gap really. We do see as well that width of the gap shrinks with disorder, which suggests that high enough disorder can eventually kill superconductivity, though that’s not definitive, since I think substances can be superconducting even without a gap.