**Drude Model**

First we gotta work out how the distribution function evolves…

**Distribution Function**

So we’ll start with our distribution function. Suppose everything is homogeneous – so position independent – and the scattering constant, is well, constant. Then we have, using the Boltzmann equation (see Stat Mech folder), that:



Now we want the average momentum. So let’s say we multiply both sides by **p**, and then integrate over all **p** (have to IBP on F term, and fact that equilibrium p = 0 on RHS). Then we’ll have:



Or another heuristic way to get to the same equation: when we apply an external force, the electrons will accelerate in that direction and scatter off impurities. The change in momentum during a time interval dt is going to be equal to the amount gained by the external force F, minus the amount that was lost via scattering. Now the probability of scattering is dt/τ, and the amount lost would just be p(t) – i.e., all the momentum acquired up until that point. So:



where F would be the external field forces (since internal e-e forces would cancel), and not the ones exerted by impurities (that’s modeled by the scattering). Dividing by dt we have,



Taking the small dt limit we have therefore,



Same as above. Of course, this model ignores the lattice completely. External forces would change the crystal momentum I believe, not exactly the translational momentum. And the effect on translational p isn’t as simple. Also scattering would perhaps be isotropic in k, but not in p. And so might not completely annihilate any forward progress?

**Conductivity**

If F is constant in time, then the steady state solution is:



And if j = nev = nep/m. And F = eE, then we have,



So,



It’s nice to estimate the scattering rate. Well we did in the EM folder/Metal Model (TD), and found 1/τ ~ 1011Hz. We can write the conductivity in terms of the material constants of the lattice,



where we define D = (1/3)ℓv. And if we define,



as the classical ‘density of states’ then we can write,



**Rate of Energy Loss (Power dissipated)**

We’ll note that when the e-‘s scatter off of the impurities, they drop from a higher momentum state, p, to a lower (thermal equilibrium momentum state), p0 = √(3mkT) . Thus each e- gives up an energy:



to the lattice with each collision (note that although <p0­> is 0, <p02> is not). This energy must be transferred to the lattice in the form of heat by conservation of energy (thus we would expect that the metal will actually heat up).

And we can determine the rate of this energy transfer. If the probability of an e- making a collision during the time interval dt is dt/τ, what is the expected # of collisions in 1s? I think this ought to be 1/τ. Thus we have N/τ collisions per second. And during each collision we have an energy transfer of



So the net rate of energy transfer is:



I’ll neglect the second term for now,



So that kind of works 😊.

**AC conductivity**

If rather F is sinusoidal, then the AC p(t) is obtained by substituting p(t) = p(ω)e-iωt,

F(t) = F(ω)e-iωt.



and therefore,



Setting j(ω) = nep(ω)/m, F(ω) = eE(ω), then,



and so,



which is,

