**Thoules argument**

So Thoules constructed an argument that the transport properties, and maybe more, ought to depend on a single parameter, which he argued was related to the conductance.

**Control parameter gT**

There was sought a way to determine whether the states of a system (at least up to the Fermi level) are delocalized or not, whether there is a single variable which this depends on. Thoules suggested the control parameter was one related to the sensitivity of the eigenfunctions to boundary conditions:



where δEn is the change in energy of the state induced by changing boundary conditions from say periodic to anti-periodic, and ΔEn = 1/Vρn is the average energy level spacing. For instance, the energy levels of a free 1D periodic system would be:



And if we changed boundary conditions to anti-periodic, then the energy levels would be:



So the fluctuation in energy, divided by the average energy spacing would be:



and we see fluctuations can be on same order as mean spacing. On the other hand, a localized state would be expected to show no sensitivity to the change in boundary conditions (well at least in the thermodynamic limit, backing up from that, we can expect exp(-L/ξ) dependence perhaps). And so,



So we could expect this parameter to be a sort of order parameter for the phase transition. Another argument to that effect is to say, suppose we pair two Ld cubes together. The (2L)d cube’s eigenstates would be a linear combination of the two Ld cubes’ states. So the nature of those states would be determined by |Voverlap|/ΔEn. But now the overlap integral |Voverlap| = |<ψ|V|ψ>| would be on the order of the bandwidth (because no Voverlap would mean just an N-fold degeneracy and no bandwidth thereby). But the bandwidth is also just δEn (I guess b/c cos 🡪 sin, and so we would shift the energy spectrum over by half-wavelength).

**Connecting gT to actual conductance g**

Arguments were made that the Thoules conductance is actually the electrical conductance itself. Estimation of δEn seems kind of flimsy, but…say we change the boundary conditions. Then our former exact eigenstate will become a quasi-particle with a certain lifetime. The lifetime of the quasi particle state is related to the quasi-particle energy’s deviation from the exact energy via: δE·δt ~ 1. Now δt can be semi-classically estimated as the time it would take for the particle to diffuse across the entire sample, which is δt = L2/D(En). So we’d have: δEn = D(En)/L2. And filling this in, we’ve get:



So it would seem the conductance can be taken as the order parameter. Turns out in 1D, gT is actually proportional to the √g. But important point is that g and this are 1-1.