**Photon-Phonon Interaction**

[working in Natural Gaussian units predominantly] This is basically the same as that for the photon-electron interaction. Going to designate phonons with bqs, bqs†, operators to distinguish from photons. So we start out with, just like with metals:



and the photon field operator is (using the CMT phase convention for a, a†):



and the density, current density operators are:



**p** is the momentum operator for the phonons. There is a current here since the atoms in the lattice are oscillating back and forth; thus we have movement of charge. It appears we’ll be neglecting the guy in the δ’s for simplicity. So,



and that makes nion(r) just a constant. Plugging these in, we have:



Well the dot product will make sure the longitudinal phonons won’t be coupled to photon field…I guess that makes sense since the EM field oscillates transverse to its momentum. If we had a homogeneous, isotropic crystal, then the polarization modes ε**q**s will be perpendicular and parallel to **q**, for any **q**. This wouldn’t be so for any real crystal, but I guess if we’re approximating it as a homogeneous/isotropic elastic medium, then yes? Well if so, then I guess we can orient the **ε**qs to be parallel to **ε**(q,λ). Then after switching to Fourier components vis a vis A just as was done in the electron-photon file, we’d have:



But will have to remember that s runs only over the transverse branch(es) of phonons. Without e­-‘s, the phonon modes are only ‘optical’, which is to say that they go to a constant for small q. On the other hand, even if we’re putting e-‘s in there, and some of the optical modes go acoustic therefore, high frequency (optical) light would require absorption by high frequency phonons, and all phonon modes at high frequency have an approximately constant frequency – since acoustic and optical modes all approach these a limiting frequency. So…we can probably approximate Ωqs as being just a constant. Then we have:



[I think we need put √V in with g(q) to get the rules to come out right, and I’m factoring out the -i from the g(q) interaction because reasons below] This Hamiltonian should be exactly solvable, since it’s at most quadratic in the creation/annihilation operators. One way to solve it would be to find a canonical transformation that makes H diagonal. Another way is via the GF stuff. So we’ll pursue that route.

**Feynman Diagram Rules**

In this form, we can adapt the rules from the electron-phonon interaction. So let’s say we’re interested in calculating the (complex time) single particle GF for phonons. Since the phonon Aqs doesn’t appear here, it seems to be not worthwhile to try to calculate the GF formed by it. Rather let’s consider the GF formed from Bqs. So we wish to calculate, generally,



and of course these are given by:



where,



and,



Now let’s get the unperturbed phonon GF. This is:



We recall,



where,



Remember that boson number is not conserved and so there is no chemical potential. We can determine the complex time GF from,



Well this is the same as the usual GF, with the phonon A. So taking the FT we will come again to:



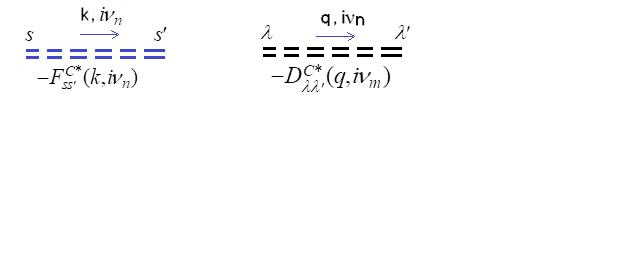
And then the photon GF:



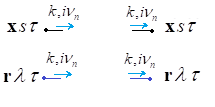
The Feynman rules for single particle GF’s are, basically without any justification…

**Fourier Space Rules**

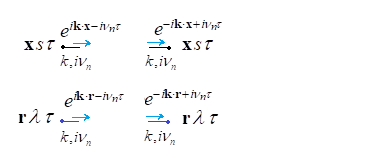
So we have our single GF’s:



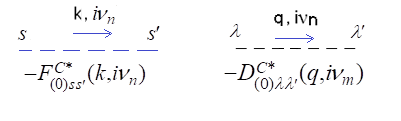
and the external points:



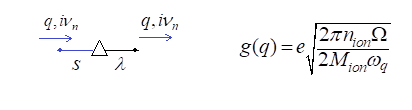
These carry the Fourier transform variable. And if we’re ultimately interested in the real space, time GF, then we must add the following factors to each:



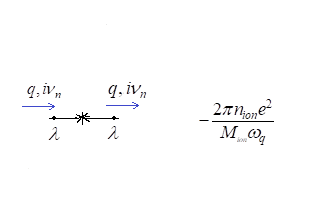
and we connect these with the bare GF’s



And we associate the Vphn-pht with:



(well, we only associate g(q) with it, but I was illustrating the energy/momentum conservation that takes place at the vertex) I think that g(q) should be associated with the vertex, and not -ig(q) because extracting a GF from the vertex requires changing B → B† and this comes at a (-) sgn cost. Moreover, this would involve at least two such interaction terms which carry with them a -i. So we have overall (-)(-i)2 = 1. So that’s why I don’t include the -i. The Vpht-pht interaction would be:



The frequency line is due to fact that Fourier transform of product introduces an extra frequency line – see that example in the Fourier transform file. Also, it might seem that momentum shouldn’t be conserved at the vertex, given the A(-q)A(q) form of the interaction term, but recall that the GF is of form AqAq† [and Aq† = A-q], and so to construct a GF, the A(-q) will have to convert to Aq†. And so then basically, the momentum flowing in will match the momentum flowing out. Same argument applies if we attach to the Aq first. Also, I’ve multiplied the pre-factor by two, because a stray photon line can connect to either end of the vertex, and so there are two possibilities associated with each vertex.

**Topology**

Connect all topologically distinct, fully connected (meaning no vacuum bubbles), diagrams together, associating with each element in the diagram the indicated term. Remember that all energy-momentum labels **k**, ω must be going in the same way w/r to the GF arrow.

**Equal time issues**

None per se´

**Signs/Numerical Factors**

I think every diagram gets a factor of one.

**Sum**

Then sum/integrate over all independent momenta - wavenumbers/energies-frequencies/indices (and polarizations, spins).



Not sure about the (1/V)·Σk deal. Since the unperturbed GF’s are presumably diagonal in σσ´, and λλ´, the internal sum over spins and polarizations will just cancel δ functions, with the net effect that whatever spin/polarization we put it in will come out.