**Thermal Properties**

Yep, here are a few thermo properties…

**Landau Free Energy Calculation**

So we want to evalue L, which we’ll recall is given by:



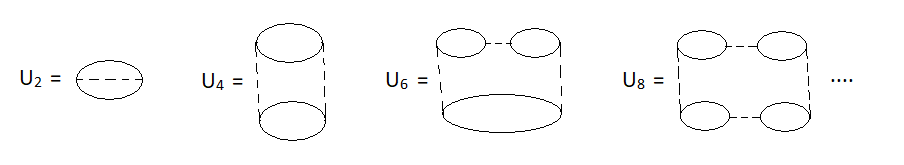
where L = -kTlnΞ is the Landau free energy. We can use the GF method to evaluate this. Our Hamiltonian is:



To start off, the non-interacting result is:



Now apropos the perturbative series, obviously we cannot sum all U’s, but we can sum a subset, much like we do with the self-energy in general. Consider the following U2, U4, U6, U8, etc.



We’ll note that the bubbles we’re connecting together are just copies of the first order term in the phonon self-energy, that made Ωq → ωq. Turns out then, that this U series similarly changes L by making the change Ωq → ωq in L0 above. Let’s see how this happens. We’ll consider the first diagram, U2. First note usually when we construct diagrams, we have two free space-time variables: (r1,τ1), (r2,τ2). But we need to integrate over these variables. Integration over the two variables separately can be replaced by an integration over the difference and over the COM. Since our usual result is a function only of the difference of these variables, integration over the COM will result in an extra factor of β and V tacked on. We made a similar argument/observation in the Fourier transform file – in that context, when integration over two points results in the same momentum conservation requirement, then we have to put a factor of V (β) back into the FT representation. And note that is the case here because if momentum is conserved at the left endpoint, then it must be so automatically at the right endpoint too. So anyway (leaving λ off of the gq as its δ3λ will implicitly force λ to always equal 3),



where ΠRPAirr is that polarization bubble thing I mentioned in the excitation file. Note the 1/2 is the Feynman factor where 1 is the multiplicity of the diagram, and 2! the factor in the Taylor series expansion. The presence of the polarization bubble could perhaps be better anticipated if we took the phonon line and raised it above the two electron lines. The second term ultimately gives,



Note the 1/4 is the Feynman factor 6/4! where 6 is the multiplicity of the diagram and 1/4! the factor in the Taylor series expansion. And in general, he alleges we’ll get:



And we can sum these to obtain,



The RHS is the correction to the TD potential. Now let’s attend to calculating the sum,



We can use the identity,



Then we have,



Now slightly modifying our definition of the phonon self-energy to this order (tacking on an η):



we would correspondingly have:



and in terms of these we may write:



and so ultimately,



This is just an intermediate result, we’ll return to in a second. But going back to the previous line:



Let’s say we find the new poles of the denominator so that we can write it:



which implies,



and then that:



it follows we can write L as:



Before proceeding, let’s introduce a frequency which depends on the coupling constant,



Then we have,



OK, now we get the summation by either recognizing how its related to the known phonon propagator (first line), or we can just do it with the usual contour integration procedure (second line)



Either way, we must now evaluate,



This can be done since,



and we obtain,



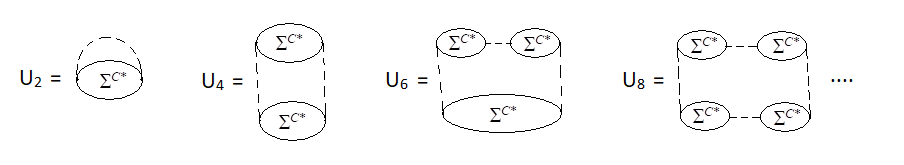
Now L0­ contains summation over transverse modes as well as longitudinal. The ΔL term contains just sum over longitudinal modes. And I don’t think the transverse guys should be affected by perturbation so that makes sense. So we have:



which is of course the form of L0, just with updated phonon frequencies! Going back to that former result,



He says there is a theorem, which asserts that this is an exact result, as long as ΣC\* and DC\* are exact. This kind of makes sense, and seems to correspond to the diagrammatic expansion:



where now we use the exact phonon self-energy [well really, I guess it should be labelled ΣC\* → -ΣC\*/2Ωq]. He says it is possible as well to reformulate our expression in terms of the *electron* self energy (rather than phonon) and the electron propagator (rather than phonon propagator). We will note that we’d be evaluating the same looking diagrams as above at each order. So that we’d have,



where this Σ is the *electron* self-energy. These two formulas are often used to calculate the GS energy in the limit of T → 0. There are two limits which are taken, V → ∞, and T → 0. It is important to take them in this order. If you reverse the order then you leave out important terms.