**Excitations**

Continuing from previous file…recall we had:



where the gap matrix is:



Δ(2)i=± are the formal eigenvalues of the gap matrix:



and |ψi(p)> are the eigenvectors. And the gap equation self-consistently determined the behavior of these eigenvalues as a function of k (and T).



where,



Note the Δi(2)(p) are implicitly functions of the various Δσσ´(p). At the end of the last file, we said that a more convenient basis to consider was the ‘coupled’ basis, defined below:



And we said solutions to the gap equation practically decouples into separate singlet and triplet solutions. Well maybe I didn’t say that, but I am now.

**Singlet Solution**

So let’s pursue the singlet (Δ00 ≠ 0, Δ11 = Δ10 = Δ1,-1 = 0) solution for a bit. The implication of setting Δs=1,m = 0 is:



I’m going to call this non-zeroy guy η(k),



This makes the Δ(2) matrix come to:



And so the eigenvalues of Δ(2)σσ´ reduce to, well,



as we can plainly see from the matrix itself. So at least that’s consistent. And the eigenvectors of Δ(2)σσ´ are obviously,



So to summarize somewhat, our eigenvalues and eigenvectors are:



Then our gap equation comes to:



where,



is independent of i. Since Δ↑↓ = -Δ↓↑­ = η(k) is the only non-zero guy, we’ll solve for *it*.



So filling in the potential,



filling in ξ, and replacing Δ↑↓ with its η(k),



Only the M0 term will survive the integration, as the M1 term is odd. So then we have:



We can presume η(k) is constant within the small energy window about the Fermi surface,



and pull out the slowly varying density of states. In this case, η cancels out on both sides, and we have:



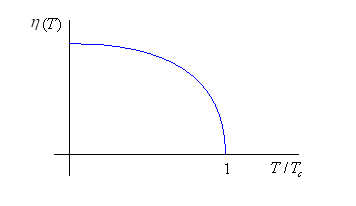
Well this is the same form that we had when studying superconductors:



We can examine the details of solving this equation in the superconductors folder. We’d expect a solution like,



and,



Anyway, won’t pursue this further, as evidence indicates that the 3He’s undergo triplet pairing anyway. But let’s take a look at the GF’s.



Remembering that the eigenvalues Δ(2)i(p) = |Δ↑↓(p)|2 didn’t depend on i, we can

pull out the resolution of identity,



which is:



Of course given Δ↑↓ = -Δ↓↑ is our only non-zero Δ matrix guy, this reduces to:



which are the very same GF’s we worked with when studying superconductivity. Moreover the F’s are spin singlet creation/annihilation operator GF’s. So that bolsters the interpretation of our solution as the spin singlet solution. Will also note from the Symmetry Considerations file, that if our H had conserved parity and spin-rotation symmetry, then we would’ve expected these to be the only non-zero solutions. So we could’ve just constructed these three GF’s way back in the beginning, worked out their equations, etc., found them to be self-consistent, solved them, and obtained the gap equation.