**Electric Susceptibility**

Gonna try to work this out a bit,

**General Features of the Linhardt function**

Now Mahan puts some effort in to working this out. For ease of reference, I’m going to put this in terms of terminology to be introduced later. I’ll borrow a definition from e-e folder:



And further, that,



where of course,



So what are some properties of this guy? First, we can see that:



Can also see that, from our work in the static case,



And we can take the q → 0 limit using,



and ρF = mkF/π2. So,



Let’s play around with this a bit,



So we can say,



with the implicit acknowledgement that we have to take limit ω =→ ω + iη. Let’s work out this term a little more.



I guess we’ll specialize to T = 0. Then,



I guess we can integrate by parts and stuff,



And so,



Now have to do a partial fraction thing. Excuse me for a second,



So can write:



And so,



Might note that if we define the unitless variables,



We can write this as, switching around the order of the ’s and ’s, and recognizing mkF/π2 = ρF, the density of states at the Fermi surface.



So the total ΠRPA is:



Luckily, this works out to what we had in the static limit if we take ω → 0. Let’s define,



where again we will have to replace ω → ω + iη, and take the limit that η → 0. Now let’s try to get the real and imaginary parts. And now we’ll explicitly do the ω → ω + iη replacement. So we’ll look at F,



where I guess we can take to be positive. And we’ll take to be positive as well. Now let:



Then for small η,



where Θ(x<0) is 1 if condition satisfied, and 0 otherwise. And we’re using the P.V. of the ln function. So,



Working a little on that inequality:



So we have:



Now let’s look at the other guy.



Now let:



Then for small η,



where Θ(x<0) is 1 if condition satisfied, and 0 otherwise. And we’re using the P.V. of the ln function. So,



Now let’s work on this inequality.



The top is always larger than the bottom. So the only way this would be negative is if the top is positive and the bottom is negative. This would require,

 and 

So we just have



It is clear that the real part of ΠRirr(RPA)(q,ω) is given by:



Filling the units back in,



So finally,



So what about the imaginary part? Well, we have:



And filling units in,



which is:



These inequalities in the imaginary part split the 1st quadrant up into five regions,

Chart

Description automatically generated

Let’s do region 1.



By same token, can see that in region 5, we have,



Now let’s do region 2.



Now region 3,



And finally region 4,



So altogether, we can say,



Probably would’ve been easier to just evaluate the imaginary part straight from the integral,



But whatever. Now let’s try to simplify ReΠ in the various regions.

**ReΠ in region 1, for large ω**

Let’s work out ReΠ a little further, in region 1, for ω far away from the εq + vFq boundary (and automatically far away from εq – vFq curve), we have:



Gonna let,



Then we have:



Then divide by ω, in those fractions,



Could write this as:



Now let’s do a Taylor series expansion. Note,



So,



Filling this in,



And going to keep just out to 1/ω4 order terms,



Grouping terms together,



This was *not* the best way to do this…too late now…



Gonna use,



Okay let’s simplify the red line,



Yay, everyone likes 0, except when they don’t. And now the blue line,



And now the green line,



Think this is correct now. So we have:



We can continue simplifying, filling ρF = mkF/π2, and kF = (3π2n)1/3,



And so, since ImΠ = 0 in Region 1, we have:



And then, to our level of approximation, we’d have:



I suspect we’d get the Thomas-Fermi result for small ω in region 1?



**What would this be in SI units?**

Well first, our ‘actual’ faux Gaussian χirr would be this one, where we multiply by 4π (see comments in the introduction of the susceptibility file in the Thermal equilibrium folder),



Then to get to the SI version, we need to add a factor of (4πε0) to some power to make the units work out. Well judging from the SI relation below, we can reason the units of the real space/time χirr, in SI, to be:



And then we take Fourier transform ∫dt eiωt ∫d3r e-ikr, which has units of TL3. So



But in our formula, the units of [Xirr]G are carried by 4πne2q2/mω2. So we need to multiply by some power of 4πε0 to get the 1/L2 units back. What will this be?



So in SI, we’d have:



Well, actually, we also need to make the units of the stuff inside the { } all come out same as 1. This will require adding in the ℏ’s we left out [we need these factors ‘cause we’re using Natural system too]. It’s obvious we need,



and εq = ℏ2q2/2m.