**Stars/Black holes**

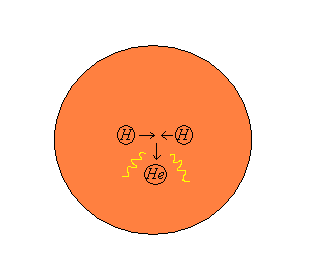
**Stellar Evolution**

Now let’s take a look at the evolution of stars, and apply some physics to the situation.

**Stage 1 (H fusion)**

Consider the birth of a star. We have a bunch of hydrogen out in space. Their gravitational attraction causes the hydrogen cloud to collapse. Gravitational potential energy gets converted to kinetic energy. As the particles accelerate faster and faster, they begin to radiate EM energy too (the H atom is electrically neutral, but I suppose that eventually their acceleration overcomes their lack of monopole moment to make their dipole moment radiation significant). So we have: ΔKE + ΔPEg + ΔEγ = 0. Eventually, the kinetic energy of the interior begins to get appreciable (could describe as very high temperature) on the inside, and fusion begins to occur. This releases a lot of energy, Eγ. The radiation pressure from these photons halts the continuing collapse, and we get a stable structure. The highest temperature and density are in the core, and these diminish towards the surface. This whole process may take on the order of 1 million years.

The H-H fusion reaction produces Helium which, do to its larger mass, sinks towards the center of the star. Surrounding it, we have the H atoms undergoing fusion itself. The photons released from the reaction Compton scatter through the interior of the star, losing much of their initial energy content. By the time they get to the surface, they’re usually in the visible light range. Neutrinos released by the interaction, however, rarely interact with matter, and travel close to the speed of light, straight through the star into space.



So to recap, H gravitationally collapses, accelerating until release of γ rays because so high that kT is enough to allow fusion, which releasing a whole bunch of γ’s themselves. The collapse stabilizes at the point where the outward radiation pressure becomes equal to the inward gravitational pressure.

**Stage 2 (Heavier element fusion)**

When most of the hydrogen has been converted to Helium, gravitational collapse begins again. This time, perhaps the He elements will be fused, resulting in a similar process, etc. Apparently one can get fusion all the way up through Carbon, and Fe. But at that point fusion will cease to be an exothermic process. Then the star has used up all its energy and will either turn into a white dwarf, contract into a neutron star or black hole, or throw off all its matter into a super-nova.

To start with, however, as heavier elements begin fuse, the radiation pressure increases due to more energy being released. This causes the star to expand outwards to 10 to 100 times its original size. The energy density decreases, and the star glows a dull red. Now it’s a red giant. As aforementioned though, eventually it runs out of this energy too, and gravitational collapse sets in. Complete collapse is prevented by the Pauli-exclusion principle. It turns into a Fermi-soup of particles. Its energy can be written as:



The first is the Fermi-energy of the ground state. The second is the gravitational potential energy of a sphere. The third is the thermal energy from fluctuations about the ground state (the star is not at T = 0 of course). The last is energy of all them photons (k is Boltzman’s constant). Assuming that the last two terms are negligible (because we’re assuming that the fusion reactions are few), we can compute the radius to which the star will collapse. Minimizing E with respect to R, we get…well first let’s work E out a little more,



And now,



Can simplify I suppose. Write M = Nmn,



So,



which corresponds to a radius equal to Earth’s orbital radius, for the case of our Sun. But I am a little confused by the validity of this calculation. We should, all things being equal, maximize entropy, not minimize energy. This amounts to the same thing when we can write E = Emech + Eint(S,V,N). Then we have S(E-Emech­, V, N), and if we want to maximize S, subject to constant E, then we minimize Emech. But this E in the highlighted equation above is not Emech. It’s Eint, really, if anything. Well, now that I think about it some more, I guess we *can* say that S = S(E – EF – PEG, V, N). For instance, the Fermi gas’s entropy would depend on energy only through (E – EF), since EF is the ground state energy with 0 entropy. So only energies above this would matter to S. Additionally, the entropy shouldn’t depend on PEG, in the same way the S for a Van der Waals gas was independent of the potential energy term. So then, to maximize S, we really would want to minimize EF + PEG, since generally, S should get larger as its energy argument gets larger. So we’d want to make E – EF­ – PEG as large as possible. But now, what about V? Because maximizing S by minimizing EF + PEG kind of presumes we’re holding V constant. But V is changing. Can we say that S doesn’t depend on V, or depends on V only through EF and PEG?

**Stage 3 (collapse)**

If the star isn’t super-massive, then I think it will shed its outer layers in a super-nova.

For massive enough starts, EF­ will be large enough so that there will be enough electrons thermally distributed into high energy regions, allowing for an inverse β- decay reaction:



This will eventually remove the electrons, allowing further gravitational collapse into a neutron star. The new radius is given by the above, but replacing me by mn. mn > me so the new radius is smaller of course.

If the star’s mass is past the Chandrasekar limit, and it’s radius, R, is less than the Schwarzild radius, then quantum and classical mechanics fail it seems. And it looks like quantum mechanics can be violated, the Pauli-exclusion principle isn’t enough to prevent further collapse of the star, and a black hole is formed.

**Hawking radiation**

Virtual particles/anti-particle pairs are created out of the vacuum all the time, but they typically disappear again – allowing conservation of energy to be preserved. But if it happens near a black hole (near the event horizon) then one of the particles may be sucked in by the black hole and the other escape. The black hole’s gravitational field would lose energy in this case to compensate. Thus a black hole may appear to emit radiation. The radiation emitted ~ 1/M, where M is the mass of the black hole. Thus tiny black holes will usually radiate energy quite a bit (and thus evaporate).

**Example**

In the future, humanity has encased the sun in a Dyson sphere and taps the unused solar power to create laser-pumped micro black holes. The Hawking radiation emitted from these micro black holes is used as a battery to supply power for space habitats and as an energy source for interstellar exploration. If such a micro black hole was created with a lifespan of 200 years, what would be its initial power output? Select the answer that comes closest to the correct value, and provide a brief explanation of your reasoning for the choice.

A black hole has temperature:



where M is its mass, presumably. And it radiates energy according to this temperature, like a black body. So,



And we know P = IA. Using the fact that the radius of a black hole is twice its mass (in Plank units – so says Wikipedia), we can obtain the following formula for the power radiated.



There is also a formula for the lifetime of such a black hole,



where the mass of the Sun is Msolar = 2×1030kg. So if a black hole had a lifespand of 200 years, then the mass would be:



and its power output would be:



**Example**

We want to derive a formula for the entropy of a black hole, as well as its temperature. Consider a spherical black hole with mass M, and radius R (the event horizon). The relationship between R and M is given by,



the Newtonian formula for escape velocity. And a black hole’s energy is presumably given by E = Mc2. Its entropy is:



Let’s see if we can get T. Well, we can get A in terms of E. So,



Therefore,



Then the temperature should be:

