**Random RG notes**

**RG in Quantum Field Theory**

He starts by relating what RG means in QED. In QED, there are parameters of the system, like e- mass, charge, photon mass. These are called the coupling constants. Physical observables, like the scattering amplitude of 2 e-‘s can be given as a power series in α, say the coupling constant of the e-γ interaction (α would be some combination of the parameters of the problem – can’t recall what it is exactly at the moment). The coefficients are given in terms of integrals over k. These integrals typically have UV divergences for any non-finite cutoff.

So he says that what you do is impose an upperbound, Λ, on the integrals. But then you get answers dependent on Λ. But he says that for certain theories (renormalizable ones) it is possible to redefine the coupling constants as e(Λ), m(Λ), α(Λ), etc. so that for any value of Λ, the answer is independent of Λ. The ‘group’ part of the name RG refers to this symmetry, that you can change the upperbound by a factor s to Λs, and the coupling constants by the same factor m(Λs), and you get the same results back. It seems that this does give physical results, since you basically do the same via different route when identify the physical mass, charge, coupling, etc., as the poles of the respective GF’s to each order in PT. Anyway, so this shows that the parameters of the system actually depend on position/wavevector.

So one wants to find these coupling constant flows α(Λ) to figure out what coupling constants to use for a particularly chosen upperbound Λ on your k-integrals. Usually you want to know to use an upper bound of Λ = ∞, b/c its easier, and so then you want to find out what α(∞) is therefore. As an example of the notation, we often write (though QFT people use the opposite sign on t),



and instead of asking for g(Λ) (g is another name for a coupling constant). We ask for g(t). In particular we define,



In Yang-Mills theory, we have that,



which has the solution,



and so for large Λ (t → -∞), we have,



In QFT, one only wants to know how the coupling constant goes as Λ → ∞, b/c there is no natural cut-off. But in CM, the natural cutoff is Λ = k = 1/a, where a is the lattice spacing. So this rational for RG is absent in CM. But there is another reason we might want to do these things.

**RG in Condensed Matter**

We can study the physics of the system at different scales by successively integrating out larger and larger portions of the system. We might want to know if particle-hole excitations or density waves are the dominant excitations in low energy scales. For instance, the physics at the microscopic scale is determined by the bare H. If we want to know about the interactions on a more macroscopic level, then we integrate out the small x part of H until we reach the scale we want. As we do this, coupling constants of the various terms in the bare H will change, and perhaps new terms will be added? Often times the new terms turn out to be irrelevant. It is possible that a coupling constant will grow until we reach a certain critical scale, at which point it will decrease in some sense. Such behavior happens when the coupling constant is marginally relevant? Well, a marginal operator is one that neither grows nor decays as b increases. Anyway, this seems to happen with superconductors. The attractive δ(x) approximate interaction grows as we scale out the higher modes, but then it decreases. The scale at which the crossover occurs is the scale at which cooper pairing takes place.

Relatedly, it can enable you to determine which FD are dominant at a particular energy scale. For instance, RPA is found to be sufficient for small q excitations. And this would be a clue towards a necessary self-consistent approximation, or towards a simple perturbative approximation.

We can study effect that a perturbation has on the low energy states. If we have an H = H0 + V which describes a fermionic system, often we’d like to know whether we can study V perturbatively. A measure of the viability of this approach is supplied by RG. For instance, I think that we expect the low k-modes to be the low energy modes. The smallest energy excitations correspond to the smallest k-values, or longest λ’s. When we renormalize the system therefore, we’re looking at an effective H which describes just the interactions between these low modes. If V grows under rescaling, then it is the operator which becomes critically important in describing these interactions, just as φ4 becomes critically important in describing the critical regime (the long λ physics). So in the same way we cannot study V perturbatively because the closer we get to the smallest modes, the larger φ4, V becomes. In one trivial case, a relevant quadratic perturbation can just lead to a shift in the chemical potential. I think that what he means here is that we cannot use PT at any finite order to study the problem. So for instance, we might expect that the e-e interaction scales down in 2D, 3D, but up in 1D? Rather we will have to sum an ∞ # of Σ diagrams somehow. Often this is done via a self-consistent approximation – like the MFT approach to solving the GF’s in the BCS theory. Other possibilities would be a variational approach

**T = 0 System of Interacting Fermions**

So now we move to fermions. He assume the typical dispersion



We’ll note that the system is gapless at present – there is no ‘macroscopic’ finite (gap between the ground state and first excited state). And we would like to use RG to analyze whether or not the system will remain gapless when we add a perturbation. We’d also like to know how best to characterize the low energy modes of the system.