**Electric Susceptibility**

So recall in the Free Day/Electrons/Nonequilibrium/Electric Susceptibility file the discussion we had concerning introducing some free charge density to the vicinity of a material, and then measuring/observing the resultant charge density that it induced in our material. We wrote it in terms off overall potential itself. We said,



and then via Maxwell’s equations we were also able to deduce that:





etc. We calculated the dielectric response of electrons in a metal in the Free Day folder, and the e-e folder. Now let’s include the phonons as well, and do it for the entire metal. So say we introduce some external (free) potential into our metal, then we have:

 (1)

where ε(q,ω) is the dielectric of the entire metal. We can relate this to the dielectric of the electrons, and that of the phonons/ions separately. So consider just the electrons. From their point of view, the ‘free’ is the actual free charge distribution, and the ions’ charge distribution. So we’d say,

 (2)

If we consider just the ions, then from their point of view,

 (3)

We can use these definitions to solve for the total ε(q,ω). Plugging the first into the second,



and first into third,



And then these, plus the first, into,



we have:



So we find,



Having in mind that V(q,ω) = Vf(q,ω)/ε(q,ω), let’s take the inverse of this equation:



Let’s fill in the small q values of εe (TF approximation), and εion. So recall from the Free Day folder, that we have:



Filling these in, we find, in small q, large r limit,



So in that limit we have:



We’ll recognize the ωq as the new screened acoustic modes of our metal. And so the metal will screen a bare Coulomb potential to look like this:



which is basically a spatial delta function interaction in space, with a sign depending on frequency. Interestingly, it’s negative for low frequencies. This is the result we got for the effective interaction in Interaction file.