**Shapiro 1**

Makes point distribution depends on dimensionality only, in case of β = 1. He does an MK scaling approach, and asserts that the insulating behavior is essentially 1D (a presumption embedded in here somewhere that will give rise to lng-normal distribution in the insulating regime). Relatedly, he asserts that transverse anisotropy won’t matter, and so we can presume the conductor to scale in quantum fashion longitudinally, but classical, transversely. So consider a 1D chain. Then we have:



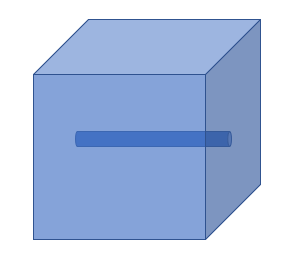
From this equation he derives:



Now he combines the two to get:



In retrospect it seems that this is done because the average, β, might very well depend on the transverse length L – seems likely enough since resistance = z/σLd-1. Now it seems we’ll presume a compositional scaling law. Now, in preparation for scaling, it appears we embed our present wire in a dielectric cube of the same length:



and then use this as the basis of our scaling process. So if we doubled the length of our conductor, then it would consist of a cube two blocks long, i.e., of length 2L, and it would have a cross section of 4 blocks. So if we increase the length from L to L + ΔL, we will have (L+ΔL)2/L2 = (1+ΔL/L)2 wires in the cross-section. In 2D, we’d have (1+ΔL/L). So generally, (1+ΔL/L)d-1. So then let’s say that our extra wires are identical – they all have the same resistance.



if resistors are identical. Then the total resistance would be R = ρ/bd-1, where b = 1+ΔL/L. And the probability distribution of resistance would be, for generic b:





Now we want the scaling (differential) equation for P(N)(ρ) as we increase the length from L to ΔL. So let’s consider.



PN is the same as P1 to first order, and so we can say:



But then it seems that we exchange ρL(1) and ρL, to write (why is this OK?):



Note it’s a total derivative, as necessary. Now he says that we can get a closed equation by working out the development of . We just multiply both sides by ρ and integrate. Then he gets:



Looks like Suslov does this differently, in a manner similar to how we obtained the PN(ρ) differential equation. So he notes this last equation has a real solution only for d > 2, which corresponds to the mobility edge, which I presume just means, corresponds to the resistance when disorder has *just* shut conduction at the Fermi level. Perturbatively, we have to first order in ε = d-2:



So,



I guess we can say that in d = 2+ε dimensions, the amount of disorder needed to localize the sample is small, and so ℓ would be large, and so the conductance would be great, and resistance therefore small. Then for ρave < ρcrit, we will scale towards insulator, and for ρave > ρcrit we will scale towards metal. Now let’s look at the distribution function. We would want to show that under these same conditions, the metal’s P(ρ) will scale towards insulator/conductor distributions. Well, let’s first look at the critical distribution. The stationary state corresponds to setting ∂lnP/∂lnL to zero. It doesn’t appear to me, however, that ρave = ρcrit *forces* this maneuver, however. Surely the average resistance must be constant, but it seems that the distribution could still evolve, while keeping the average constant.



We must set C = 0 to make the distribution normalizable, and then Cʹ to something to normalize it. He gets:



In particular, we have:



Then, it appears he says that when ρave > ρcrit, the resistance will generally scale upwards, and when ρave < ρcrit, it will scale downwards. So then keeping just the largest terms in the distribution equation, in these circumstances, leads us to the following results.



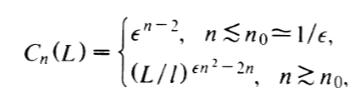
and A, B are just some constants. But note again, that it does not appear ρc emerges from P(ρ) itself, but only from the ancillary ρave equation. The insulating regime stuff seems right: both lng and lnρ have ln-normal distributions I believe. But what about metallic regime? If we say ρ = 1/g, then P(g) would be … strange.



This is a legit distribution, but has an exceedingly long tail. The average conductance wouldn’t even be well-defined, nor the variance. So we don’t get UCF. So it seems that these results are not quite kosher.

**Shapiro 3**

Previously he found an equation for P(g) in the various regimes. Pconductor(g) seemed quite off, Pinsluator not to bad, but ln-normal, and Pcrit was Cauchy-like. Looks like in the interim, these cumulants were calculated using NLσM,

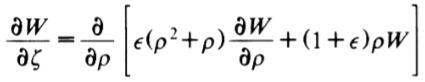


Now it seems he is going to try to calculate the moments of the distribution Pcrit(g) in d = 2+ε dimensions, and he’s trying to see if he can reproduce them. He mentions that these results are still consistent with a limiting universal distribution at the critical point. One example distribution Pε(L,g) would be:



L0 could contain information like ℓ, kF and such, and so the distribution might only become universal in the large L limit. Mentions that, physically, for a given realization of disorder, the distribution P(g) is thought to become universal in the large L limit (only large L because then the CLT will have taken over basically, and washed out microscopic details of the Hamiltonian). Also says that the limiting distribution ought to be even independent of ℓ at the critical point…why? Well, I guess ℓ will have been tuned to the critical value anyway.

So he says he got following PDE for P(g) at the critical point (d = 2+ε dimensions), in Shapiro 2.



where ζ = lnL. How does this compare to his earlier result?



and d = 2+ε, ρcrit = ε/2,



So yeah, this is close to his earlier result. He mentions that any initial distribution will scale towards the limiting one, where ∂/∂ρ = 0. He takes the PDE, integrates w/r to ρn and writes down a recursion relation for the moments:



He notes that these will converge to L-independent values for n < 1/ε,

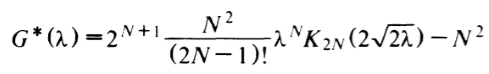


and then diverge with L for n > 1/ε.

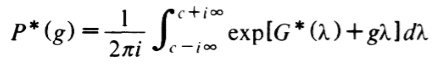


These divergent moments also depend on the initial probability distribution for their prefactors, he says. These results are very close to the NLσM results, but the exponent is a tiny bit off. But worse, is that these results are for ρ-moments; it would seem that g-moments would have completely different behavior.

Next part of the paper is to try to reconstruct the probability distribution off of the NLσM cumulants. He says that it is, mathematically, strictly impossible to reconstruct the critical distribution from the moments cited in the NLσM, because they diverge with n, faster than n! in particular. And so I guess the moment generating function doesn’t converge or something, and so we cannot construct P(g). But he says we can make ‘plausible’ arguments to work it out (kind of like Borel summation I presume), and under these conditions is able to work out a result for the moment generating function,



and also for Pcrit(g),



He finds a delta function at the origin with amplitude ~ exp(-1/ε2), indicating a super small fraction of the samples would be in effectively insulator configurations. Then approximately Gaussian around the expected g = 1/ε, and then for larger g, power law decay.