**Excitations**

Now let’s take a cursory look at how disorder affects the excitations of a superconductor. We’ll start with the mean field Hamiltonian, basically same as the last H, but now we’ll employ MFT from the beginning. So,



where,



and add disorder to it,



where, as we’ll recall from the Metals/Electrons/Impurities file,



Though the Hamiltonian is technically solvable exactly since it’s bilinear, the disorder average we have to take makes that unfeasible. So we’ll attempt a perturbative expansion of the single particle GF.

**Nambu Formalism**

So we can group these GF’s into a matrix form that proves convenient for some calculations we’ll do later. We define:



(k is a vector too, but whatever). Note each component of α destroys eliminates net momentum k, and eliminates net up spin angular momentum. Each component of α† adds net momentum k, and adds net up spin angular momentum. Plus, these operators form the basis of the γk and γk† operators we discussed in the Excitations files. But αk, αk† comprise different linear combinations of those operators than the γk, γk† guys. Anyway, so forming the GF:



If we take the Fourier transform, we have:



The 22 matrix element follows from fact that (where we’re changing variables in last term in top line):



and explicitly, this is:



where we take Δ to be real, and recall the Pauli spin matrices,



It’s worthwhile to note that we can write this as:



, where 1/ is to be interpreted as matrix inversion. Just to check,



where in the third line we use the fact that the Pauli matrices anti-commute. We’ll also use, later, the spectral representation of **G**,



Working out Ak(x), using 1/(x±i0+) = P(1/x) ∓ πiδ(x),



Assuming Δ is real still.

**Feynman Rules**

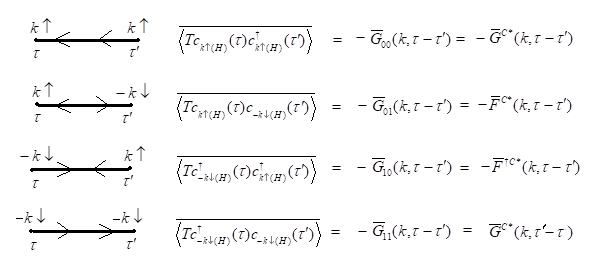
So we’ll use as the interaction picture the mean field BCS Hamiltonian, and expand the GF in powers of the disorder potential. Inevitably, the perturbative expansion will involve the set of four Nambu GF’s previously discussed. So we’ll represent the disorder averaged GF’s via:



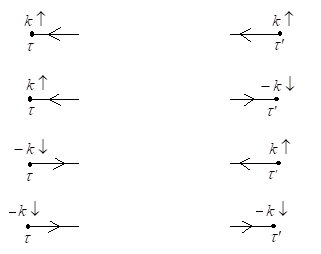
whose Fourier transform is:



and would be represented with double, or bold, lines I guess. Well, I guess to keep in accordance with how I’ve expressed Feynman rules in past, I’ll state the rules for the time-ordered correlations themselves, not the GF’s per se´. So:



(note left arrow is always k↑ and right arrow -k↓) And the perturbative series works as follows. So we start with the external points. We represent the two arguments/operators in the GF we’re trying to get with external points. We use the first pair if we want G00, the next for G01, the next for G10 and the last for G11.



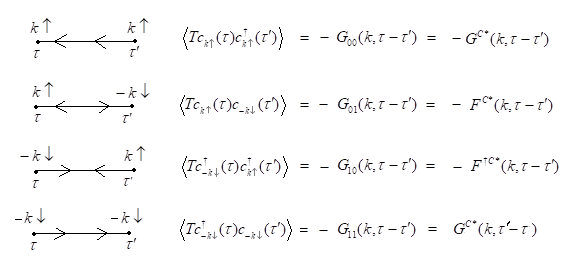
At each order of perturbation theory we bring down the same number of vertices and connect them to the external points, and each other. The ‘unperturbed’ GF’s would be the full Mean Field BCS Green’s functions (no disorder),



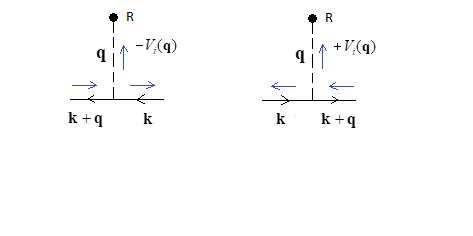
whose Fourier transforms are:



and represented by skinny lines (and again, I’ll be taking the lines to represent simply the time-ordered correlation functions, rather than the GF’s per se´ - difference is just minus sign),



Then we’d connect these GF’s with impurity vertices (note that Green’s functions can now connect to either end). I’m guessing on this part, but looks like the spin sum on the disorder interaction will collapse onto whichever spin is being carried by the bare GF leg connecting to it (and since interaction conserves spin, the line heading out of the vertex will carry the same spin). Also, since bare GF lines can connect to either end of the disorder vertex, I think we end up effectively with two vertices. The conventional vertex is the guy on the left, and the unconventional one, the guy on the right. The unconventional vertex is unconventional because connections to it would have a minus sign compared to the same connections to the conventional vertex, because (I think), if we just switched the order the vertex creation/annihilation operators, the connections would become conventional-like, and transposing operators costs a minus sign. That’s my theory anyway. So the vertices are:



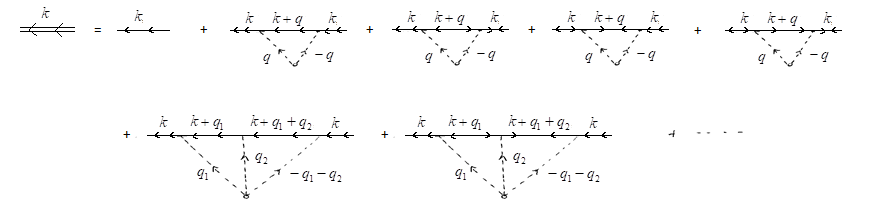
The eiq·R that would be associated with each external point in the vertex gets integrated over ∫(1/V)d3R, and so instead effectuates momentum conservation at the external leg, as we’ve seen before for disorder averaging. Also, we presume Vi(0) is either absorbed into the chemical potential, or zero. Each impurity dot will also ultimately get a factor of ni, where ni is the impurity density Ni/V.

And sum over all independent momenta, q, via (1/V)Σq, or ∫d3q/(2π)3 in the continuum limit.

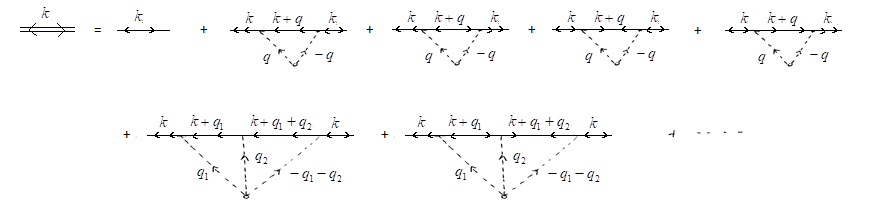
Disclaimer: we could associate the lines with the GF, instead of just the time-ordered correlations functions, i.e., negative the GF’s. Then we’d correspondingly change the signs associated with the V’s to.

**Working G00 out to first non-zero (i.e. second) order**

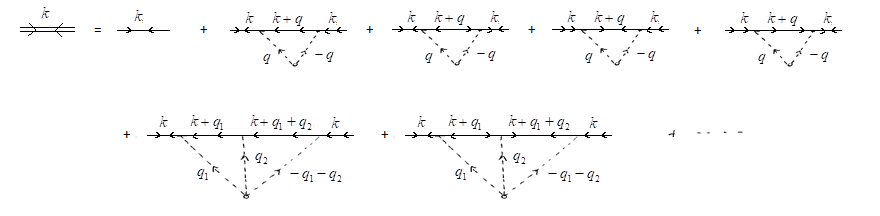
Based on the above, we can represent 00C\*(k,iωn) via a diagrammatic expansion (momenta in GF lines are flowing left → right):



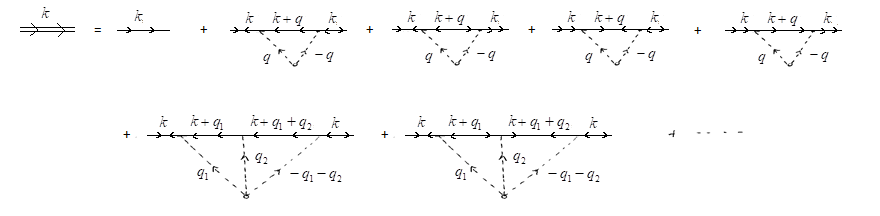
and 01C\*(k,iωn)…



And 10C\*(k,iωn)…



and last, 11C\*(k,iωn):



Let’s work out 00C\* to second order (first non-zero order) straight from the equation and compare to the diagrammatic prescription. So,



where,



The second order stuff gives us:



where factor of two is because equivalency of contractions whereby we simply switch s1 and s2, Now applying momentum and spin conservation,



Continuing,



Surprising that the q2 = -q1 condition was effectuated before the disorder average. Well, now let’s disorder average via (1/VNi)∫ Πjd3Rj for each set of coordinates. Only when j1 = j2 will we get a non-zero contribution, which will be V2Ni. And there will be Ni cases of this. So we’ll have:



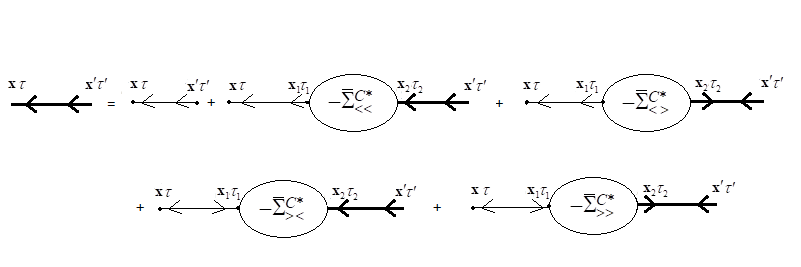
and putting in terms of GF’s,



and this matches the diagrammatic expansion, according to the associations given above. It’s alleged that we do the diagrammatic expansion for all GF’s at once by using the Nambu matrix GF, but I’m not going to think about that.

**Self-Energy Equation**

It seems to me that we can write out a matrix self-energy equation, similar to what was done for interacting identical particles in the Quantum Mechanics/Identical Particles/GF-Perturbative Expansion (V1,2) file. Instead of + - vertices, now we’d have < > vertices. And, borrowing from that file, we’d have something like (I know we didn’t really put the Feynman rules above in position space, but)



where Σ<> refers to all non-cutable diagrams that begin with a < vertex and end with a > vertex, etc. And we could write this out mathematically as,



where



After the disorder average, our system is homogeneous, and so everything will be a function of the difference of position arguments, and difference of time arguments. So then we can take the spatio-temporal Fourier transform,



We can solve for **C\***,



which I’m going to write as:



but of course 1/ means ‘matrix inverse’. We can fill in our result for this to write,

