**Thermal Equilibrium Properties**

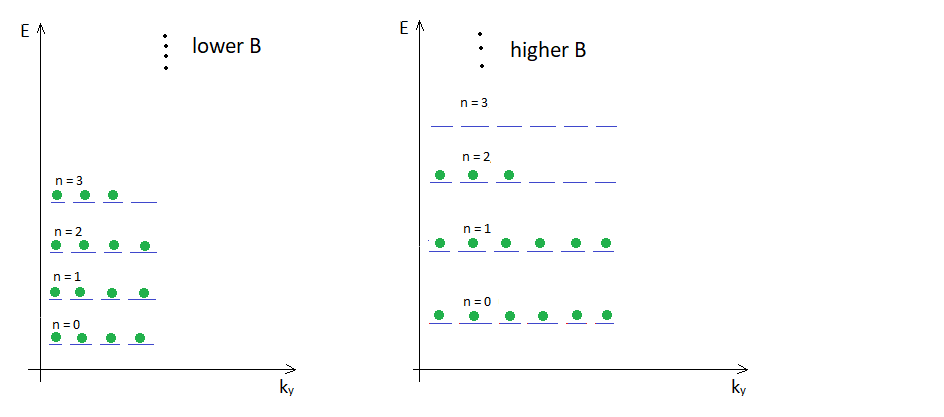
We’ll take a look at a variety of thermal properties…

**deHaas-VanAlphen Effect (2D, and T = 0)**

In our diamagnetic calculation in previous file we took the limit that T ~ room temperature, and B small so that kT > ωc. If we take the opposite limit, that of relatively high B, then turns out we will see the magnetization undergo periodic oscillations in (inverse) magnetic field strength. We could try to work out this out from expression for L. But that seems hard. So going to back down to 2D for one, and then also going to T = 0. Then in the Ground State, we have electrons filling many of these Landau levels, with energies,



As increase the field, the levels rise and their degeneracy increases. So the GS energy, say, EGS(B) will tend to go up because of the former effect, but go down too because the increasing degeneracy will cause electrons to drop from higher levels to newly created lower levels. Of course eventually the lowest level will have a degeneracy so large that all e’s fit inside it. Energies depicted below for increasing B, and again, just going orbital motion (can do the calculation with spin, but would have to do each spin separately, and I think we’d find a variety of behaviors then, not necessarily periodic as the two sets of electrons may actually cancel the other out).



Let’s try to get the energy of the ground state as a function of B. So say we have N particles. Let ν be the so-called *filling factor*, which counts the number of Landau levels occupied. Of course levels need not be completely occupied. And so ν can be fractional. So for instance in the first diagram we’d have ν = 3.75, and in the second diagram ν = 2.5 rather. We can write the number of particles and energy in terms of the filling factor. So,



where BA|e|/2π is of course the degeneracy of each Landau level, and the length ℓB was introduced in the excitations file, and extra factor of 2 is to account for spin degeneracy (so 2BA|e|/2π is that degeneracy of each level I guess I’m saying). Now let p be the last completely full energy level. p = 2 in the first diagram, and p = 1 in the second. Might note that ν = p + 1 + λ, where λ is the fractional occupation of next energy level. λ = 3/4 in the first diagram, and 1/2 in the second. Parenthetically, this means we can write λ = [ν-1] where [x] is the greatest integer less than or equal to x. So we can write the energy as:



Now putting in terms of ν,



Now using,



we can say,



Now let B1 be the field strength at which ν = 1, i.e., all particles are in the ground state. Then we have:



What is this field strength? Well in the QM folder/Symmetric gauge, we calculated this and I think was B1 ~ O(104 T). So really high. Anyway, it follows, upon taking the ratio of the top two equations, that:



So we can write:



Now using the third equation, B1 = 2πN/2A|e|, we can say,



Now recognize the GS energy of the 2D electron gas with no field as (see Excitations file):



where n = N/A. So then we have:



λ is a periodic function of B, which oscillates between 0 and 1. And then recall we said that λ = [ν-1], and ν = B1/B of course. So we have:



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Makes sense that when B = 0 we should get the normal electron gas GS energy, and we do. Can also see periodic oscillations in inverse field strength. Period is clearly 1/B1 and this makes sense, since the λ thing should repeat itself every time B1/B – 1 advances by 1, meaning:



So we have for our period:



This is, in other words, simply the change in field strength required to change ν by Δν = 1. Well the magnetization would go as M = -∂EGS/∂B. So the magnetization will also be a periodic function with same period. Turns out other quantities are too, like the resistance, a phenomenon referred to as magneto-resistance. These oscillations are washed out when kBT > ωc, as then we don’t get such a sharp distinction between occupied and unoccupied levels.

**Semi-Classical deHaas-VanAlphen Effect (3D, T = low)**

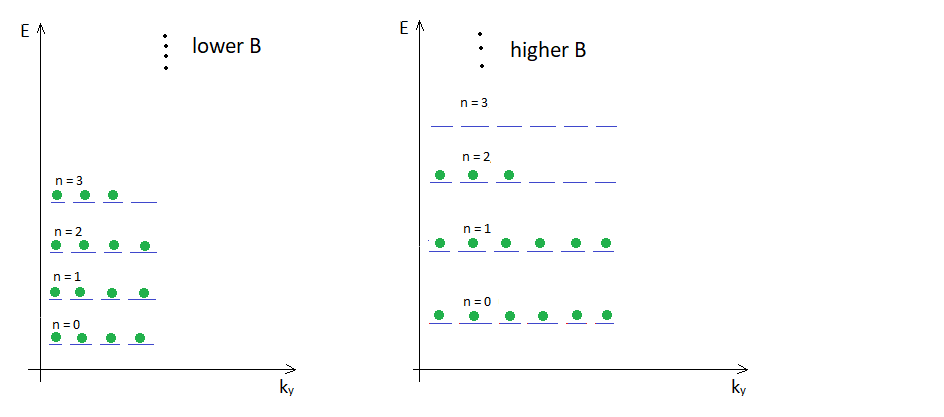
Note we can write:



In our diamagnetic calculation before we took the limit that T ~ room temperature, and B small so that kT > ωc. If we take the opposite limit, that of relatively high B, then turns out we will see the magnetization undergo periodic oscillations in (inverse) magnetic field strength. We could try to work out this out from expression for L. But that seems hard. So going to back down to 2D for one, and then also going to T = 0. Then in the Ground State, we have electrons filling many of these Landau levels, with energies,



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Let’s try to get the energy of the ground state as a function of B. So say we have N particles. Let ν be the so-called *filling factor*, which counts the number of Landau levels occupied. Of course levels need not be completely occupied. And so ν can be fractional. So for instance in the first diagram we’d have ν = 3.75, and in the second diagram ν = 2.5 rather. We can write the number of particles and energy in terms of the filling factor. So,



where BA|e|/2π is of course the degeneracy of each Landau level, and the length ℓB was introduced in the excitations file, and extra factor of 2 is to account for spin degeneracy (so 2BA|e|/2π is that degeneracy of each level I guess I’m saying). Now let p be the last completely full energy level. p = 2 in the first diagram, and p = 1 in the second. Might note that ν = p + 1 + λ, where λ is the fractional occupation of next energy level. λ = 3/4 in the first diagram, and 1/2 in the second. Parenthetically, this means we can write λ = [ν-1] where [x] is the greatest integer less than or equal to x. So we can write the energy as:



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