**Conduction**

Can’t talk about linear acoustic spectra here, and the concommitant characteristics this imparts to the temperature dependence of the conductivity, since that involves the phenomenon of screening, which requires electron-electron interactions. We will do this in the electron interactions + phonons folder. So I’ll just do this.

**Conductivity via Kubo Formula, kind of**

Just a sketch. So we start with our electron – phonon Hamiltonian,



and add an electromagnetic field to get, ultimately:



The last term we’ll call VEM. And note definitions:



(ρ is charge density, and means -i∇) Our non-equilibrium steady-state distribution function is:



where Ueq includes all terms in H sans VEM. Now we calculate the expectation of the current,



And so ultimately,



where,



The steady state result would be obtained by taking t0 → -∞. So we’ll do this. We should have homogeneity, so then a Fourier transform on time/space would give us,



Now we can extract the conductivity tensor. In our gauge, if we have an electric field then it would be given by:



So now we have:



So we have that the conductivity tensor is:



and specializing to DC,



and expecting a real σDC, we have:



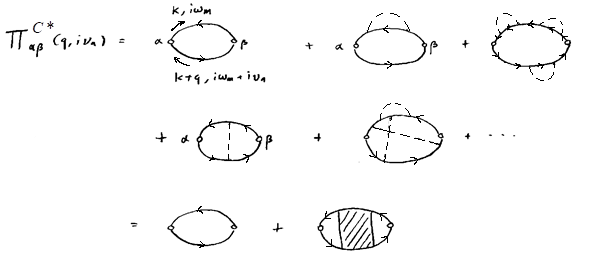
Presuming isotropy, then even more simply,



The current-current correlation function would be given by, as before, without disorder averaging though, because we don’t have any impurities here….



which amounts to, diagrammatically, a structure very similar to what we had before.



Basically just replace the impurity vertex with the e-ph vertex. Otherwise, the Feynman rules are the same.

Construct all diagrams with one phonon, two phonon, three phonon vertices, etc. placed on either of the two legs.

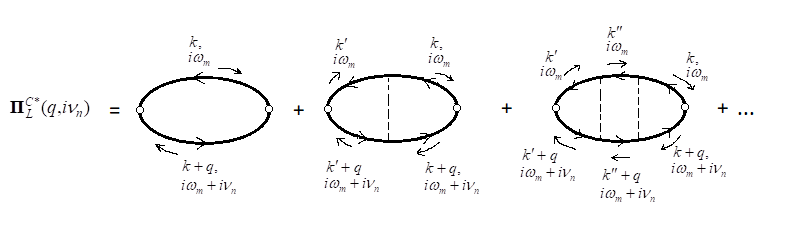
Start with ‘dummy’ energy–momentum (k,iωn) coming out of the open circle (bare current vertex) into the top line. Note that this won’t necessarily be the momentum going into the bare current vertex on the right from the top line. Impose energy-momentum conservation at each vertex. Add (q,iνn) ’boost’ once cross the other open circle on the right hand. Overall energy-momentum conservation will ensure that energy-momenta ‘below’ the L and R vertices will be (q,iνn) greater than the momenta above the L and R vertices. Each open circle (called the bare current vertex) labeled α connecting unbroken lines labeled k1 and k2 represents one factor of jα = eα = e(k1α + k2α)/2m - this factor is sort of an average current (charge \* average velocity) - the sum of the incoming and out going momenta multiplied by a prefactor. The prefactor comes from the prefactor in the correlation function, and the momenta come from the gradients. Each single/double unbroken line labeled (k,iωn) represents a factor of G0C\*(k,iωn) / GC\*(k,iωn­), as before. Each crossed circle represents a factor of ni and each dashed line labeled q, represents one factor of Vi(q), as before. *The overall Fermion loops doesn’t get a -1 here.*

Sum over all unconstrained momenta and (the one) unconstrained frequency iωm, with the 1/V and 1/β factor as usual. The GF’s carry spin indices, but presuming they’re diagonal in spin indices, then overall sum over spins just result in a net factor of 2 on the outside as well.

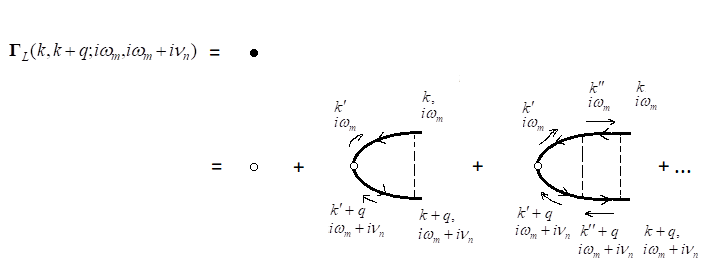
Disclaimer: Reasoning straight from a Wick expansion approach, we are accustomed to associating each fermion line/contraction with -G, and each impurity potential with -Vi. But we always have an even number total of these, ‘cause we start with two GF’s. And then each phonon line adds another GF. So we’ll always have an even number of (GF’s + phonon lines). So the negative signs are of no consequence.

**‘Drude’ Conductivity**

The similarity between the impurity calculation and the phonon calculation suggests that the exact same procedures as used before can be used to here. For instance, to get the Drude result, we would just sum all the ladder diagrams on top of dressed GF’s (i.e. GF’s that incorporate the first order contribution to the self-energy from the eph vertex).



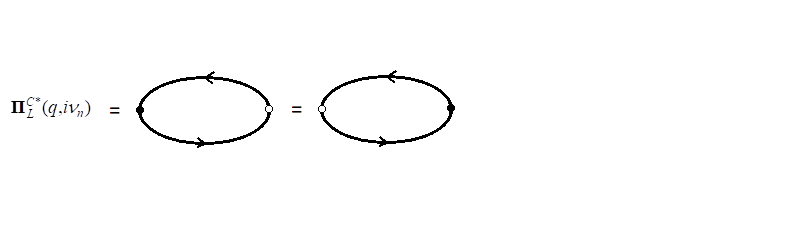
We can try to do this brute force, or by working out a recursion relation for the irreducible vertex.



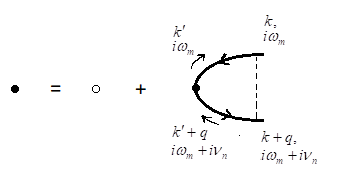
in terms of which we can write:



and diagrammatically,



Suffice to say, Mahan looks at the recursion relation for the ladder vertex,



and solves it in a manner similar to how was done in the impurity file. He ends up with a result identical to that obtained from the Boltzman equation.