**Single Parameter Scaling Theory**

So those guys: Ramakrishnan, Lee, Anderson?, etc., argued that the conductance itself ought to obey a single parameter scaling law:



where β(G) is some as of yet undetermined function called the scaling function. We would expect however that β(G) is an analytic (I wouldn’t expect that the derivative on the LHS of the equal sign had any kinks in it), monotonic function. We note that if you are at a conductance where β(G) is greater than 0, as you increase the length, G must increase also because the derivative is positive; but if you are at a conductance where β(G) is negative, as you increase the length, the conductance must decrease, because the derivative is negative. Finally if you are at the conductance where β(G) = 0, then as you increase the length, the conductance won’t change - it remains scale invariant. So there is a possibility to have a critical conductance, about which a conductor will scale to a metallic state, or to an insulating state, or nowhere.

Note, as should be apparent given the previous file’s discussion, g = g(L/ξ) is consistent with the SPS form, since:



**β(G) in different dimensions**

So for g >> 1, we have Ohm’s law, g = σ0Ld-2, and so:



Backing away from the extreme limit, we can apply our weak localization results, and our Self-Consistent Theory results to say:



The 2D Self Consistent result was apparently also found by some others using the NLsM, or some other approach? Apropos the 3D result, apparently Hikami (?) calculated β(g) out to 1/g3 order in some other fashion and found the same result. Interesting that the d = 2, d > 2 cases are so different. I think it was suggested by some(one?) that this indicated the expansion was non-analytic in powers of ε.

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On the other hand, we’ll argue that g is exponentially decaying in the insulating regime:

g ~ g0e-L/ξ, in which case,



If we fill in our values/guesses for the behavior of g per dimension, and compute β(g), it can be plotted below:



Given the plot we will argue that can linearize the scaling function near the critical point. We’ll write the general form as:



**Generic implications for insulating state**

Let’s consider g on the insulating side of the transition, and integrate from g0 close to gc out to g << 1. Note we only know the form of β(g) in the asymptotic limits, so we have to split the integral up into parts:



**Generic implications for conducting state**

Now let’s go integrate on the other side of the transition, from g = g0 close to gc to g >> 1. Then we have:



Note how the same length scale, ξ, shows up in both calculations. And we see that the slope of β(g) controls the scaling of the correlation/localization length near the critical point. Now let’s look at the conductivity, and how it goes to zero as we approach the critical point:



So we see that the transition from conductor to insulator is continuous. Note result implies the transition is continuous. So there is no minimum conductivity, though there is a min conductance, in the metallic side. Might imagine that at the min conductance, roughly (kFℓ)d-1, interference already destroys the conductivity. And we see Wegner’s scaling exponent law is preserved automatically. We’ll also observe these results congrue quite well with the self-consistent theory results for Wc, ξ(W-Wc), and σ(W-Wc), if we take gc to be e2(kFℓc)d-1, g0 = e2(kFℓc)d-1, i.e., the Boltzman conductance at the level of the m.f.p., and L0 = ℓ.

**Generic implications close to critical state**

Then sticking close to the critical point we have:



**Specialize to Self-Consistent Theory in 2+ε dimensions**

Let’s specialize to d = 2 + ε systems. Both the WL and SC theory agree on the form here, even up to d = 3. While its questionable whether this form is a good approximation near the critical point in d = 3, it should be a pretty good approximation for d close to 2, since here the critical point occurs at very large g, where disorder is weak, and calculations more assured. We’ll need to linearize around the critical point, so consider:



the critical conductance would be:



and observe that it is very large in d = 2 + ε dimensions. Observe again, how this form agrees quite well with the SCT when we substitute in the value of the conductance at the level of the m.f.p. Now linearize about this point:



So we see the prediction is ν = 1/(d-2) and s = 1, accordingly. But in fact, numerical evidence indicates ν ~ 1.5, and s as well, by Wegner’s scaling law.

**Implications in 2D**

Now let’s integrate β(g) along the length from high to low g. I’ll use the WL β(g).



If we set L0 = ℓ, and fill in g0 at the level of the m.f.p. too, we get a result for ξ qualitatively identical to the WL result, as we ought.

**Implications in 1D**

Let’s do the same thing as for 2D, again using the WL result for β(g),



Again, we find results nearly identical to the WL stuff.