**Thinking about stuff**

**Examining self-consistency of various scaling laws**

So I was thinking that I could just write any old:



and it would be valid. But it seems rather that this equation may suffer from internal inconsistencies. Problem is that certainly we could say any equation of this sort could be true for a fixed L. But the question is whether its true for ***all*** L. And for that to be the case, there are some revelatory self-consistency requirements that it must obey. For instance,



A self-consistency condition would be:



So we need,



There are other issues too. Consider that this equation seems to imply the following two conclusions:



When all these, presumably overlapping, conditions are met, then the straight-forward solution:



and the solution of:



all match. Otherwise they do not…so far.

These scaling laws they propose *may* only be valid for b close to 1. And that is why extrapolating solutions to all b doesn’t satisfy ODE. But that otherwise, extrapolating would work. Is it possible a scaling law be valid only to first order?



**Example 1**

Law:



Is equation self-consistent?



So yes. Proposed solution would just be:



Does it obey the scaling law?



So yes. Now let’s develop the ODE.



so they match. What about that last condition?



Checks out.

**Example 2**

Law and solution:



Is it self-consistent?



Yes it is. Proposed solution is:



Does it satisfy the scaling law?



So yes. Now what is the ODE?



So that works too. What about that last condition?



Checks out.

**Example 3**

So let’s try:



Is it self-consistent?



So yes. Now let’s try the old proposition:



Let’s see if it works:



This does satisfy the equation. The ODE would be:



So these are consistent. What about that last condition?



So that checks out.

**Example 4**

So let’s try:



Now let’s try the old proposition:



Let’s see if it works:



So yeah. Now what about ODE?



Let’s work it out,



Checks out again. Let’s try the last condition:



So that also works.

**Example 5**

Scaling law is:



This has proper behavior at b = 0, 1. Let’s do the consistency check:



So it fails the self-consistency check. Consider proposed solution:



Does it satisfy the original scaling equation?



Nope. hmmmmm. Let’s consider the last test,



So this test would also fail. Now consider the ODE and Solution:



And so let’s test whether our proposed solution satisfies this ODE:



Nope again. So I think the problem is simply that the ‘problem’ isn’t well-posed. Let’s consider the problem to 1st order. Is it consistent then?



So it does hold in this reduced limit. Note that we cannot propose the straight-forward solution ρL = Lαρ1L for this first-order process, because this assume we can make b large (i.e., L), when we can only make it 1+ε.

**Example 6**

Consider the following scaling law:



But what about self-consistency?



I’m going to go with no. One could propose the following solution:



Then ODE would be:



Now does solution solve this equation?



So then question is, do the following match?



Seems not. Again, I would posit that the scaling equation isn’t self-consistent.

**Difference between solutions of ∞ order scaling equations and first order ones**

So it seems that not all scaling laws are self-consistent. Like,



But perhaps all/most can be made so if we only go out to first order in b.



besides the fact that the first one doesn’t make sense? Or why does the second one make sense? Obviously 2nd is special case. If we let ε be finite, then it would gives us the 1st guy. I guess I’d like to know, in a physics context, why the 2nd equation could be true for some scenarios, while the 1st would not. Not sure, but its kind of like say radioactive decay dN = -λNdt. We can assume that changes in particle number are are proportional to changes in time, for small time changes, but not for large time changes.

Different question, suppose we have an equation like,



Does any scaling relationship follow from this? Certainly we could say:





So this isn’t of the form f(b,ρL), as we’ve seen. So these sorts of laws, which start from some fundamental point, do not necessarily scale appropriately.