**Conductivity**

Let’s examine the conductivity. A point is raised that if we were dealing with a pure jelly model, then it we shouldn’t have any resistance at all, since an electric field would just uniformly accelerate the center of mass of all the electrons. So there has to be some mechanism for scattering beyond the ee interaction. In the absence of disorder, the crystal lattice itself provides the mechanism for dissipation of momentum via Umklapp scattering. Tried to illustrate this below. There’s four points of the Reciprocal Lattice.

A diagram of a diagram

Description automatically generated

We have two electrons with initial orange k-vectors collide. The addition of the two vectors would place it outside the BZ, so we add a reciprocal lattice vector to put it back inside the BZ. The red vectors are the possible final k-vectors of the electrons. And we can see they are now pointing in the opposite direction. So in effect when one electron scatters off of another, it may indeed end up going in any which direction, thanks to Umklapp scattering. So this sort of scattering acts as an energy/momentum relaxation mechanism. We can see that this mechanism would be most pronounced at high temperatures, when the k-vectors of the electrons would be large.

**Conductivity via Boltzman Equation kind of**

So generically we have, for the evolution of the rkσ occupation number function (see Stat Mech/Classical NESM Boltzman 2PI):



**High Temperature**

If T is large, then scattering from one state to another is unimpeded by the Pauli-exclusion principle b/c any individual occupation will be exponentially small ~ exp(-ε/kT). So we can consider our model to be one electron in isolation of the other N-1. We consider these other N – 1 electrons to be impurities evenly distributed through the sample. And we can consider them to be impurities in the Umklapp sense; scattering with other electrons thermally relaxes the electron. Instead of going through the whole process, we’ll just estimate the transport scattering rate.



where |k> are box normalized states. We can estimate the T-matrix by using the Coulomb potential result for scattering cross-section. The relation between T and dσ/dΩ is:



But this equation presumes continuum normalized states, not box states, that are implicitly used in Tkq above. So to put in terms of box, we’d say,



where |**k**> is a box normalized state.



(this is same as we found in Stat Mech/Classical NESM Boltzman 1PI). And our result for the scattering cross-section is:



But again, this cross section is for the Coulomb interaction. In reality, the potential will be screened so it will be short ranged. The effect of screening is to cut the cross section off at small angles (for analogy, see the 3D scattering perturbation theory file for an example of the cross section of the Yukawa potential). So in reality we’d have something like (going to faux Gaussian units so ε0 → 1/4π)



where Θ is the step function. So then we have:



Whatever we get, it’ll be something like,



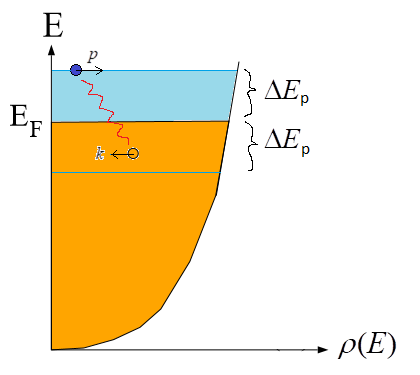
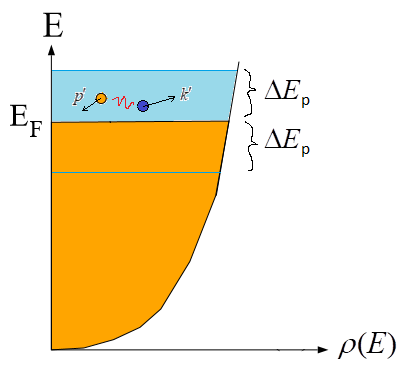
Noting that thermally vk ~ √T, εk ~ T, we should find the following temperature dependence,



So as T increases, the transport scattering rate decreases, interestingly, and ρ ~ 1/T3/2. Why should it go down? I guess higher T means higher v’s and greater chance overall of forward scattering. Now let’s examine the small temperature result.

**Low Temperature**

Now we have to take into account the Pauli-exclusion principle, and we cannot treat the collisions as 1 e- scattering against a bunch of ‘impurity’ e-‘s. At low T’s, the excited electrons will be close to the Fermi surface. And the energy window the excited electrons occupy above the Fermi surface will be ΔEp ~ T. The exclusion principle will make the scattering rate super small. For consider an electron injected with energy ΔEp above EF (red squiggle is interaction), and scattering with another one within the Fermi sphere.

If the Pauli-Exclusion principle didn’t apply, then it could scatter with any other electron within the Fermi sphere. I suppose it technically could even still, if it were to happen that the scattering event caused the two electrons to precisely exchange places. But this is astronomically unlikely (well at least it would be in 2D or 3D – but in 1D I guess there’s a 50/50 chance? which is perhaps why 1D electrons don’t act as Fermi liquids?) So for practical purposes, the two must scatter into a space above the Fermi surface. And they can only do that if the other electron is within the same energy window ΔEp below EF as the injected electron is above it. We can be somewhat quantitative. We’ll just try for the relaxation rate. So our Boltzman equation is:



And comparing to the RTA approximation,



we can see that 1/τ would be approximately 1/τ = -∂(RHS)/∂f. If we do that to our Boltzman equation, then we get, ignoring spin:



What is the matrix element? Well,



switch to c.o.m. variables,



Jacobian is 1.



For the potential Fourier transform, which would be screened, thanks to the other electrons roaming around, we can use the simplest screened model – the Thomas-Fermi approximation. We got (see Thermal Equilibrium properties folder):



But this is the potential, not potential energy, so we have:



But let’s be even lazier, and just use the q = 0 result. This should put us at least in the ball park.



Continuing,



Doing the sum over k will give us:



And the sum over k´, p´, would be restricted to area above Fermi surface thanks to 1-nF(εk´) and 1-nF(εp´) factors. So,



The nF left seems to restrict the p´, k´ sums to within that aforementioned energy window ΔEp above the Fermi surface now. So



where the Δp reminds us that the radial part of the k´, p´ integration only proceeds within that momentum window (energy window). Gonna rough it from here on out. I’ll change coordinates from **p** → εp, Ω, and write:



Now if the energy window is small, we can just approximate the magnitudes k´, p´, and p, as being kF. And then we have:



where in the last we do the radial integration, and am implicitly measuring angles w/r to the direction of **p**. Using ρ(εF) = 3n/2εF, we have:



The angular integration is just some number of order 1. And so just have:



So ρ ~ T2 it would seem, for low T. So we have:

Chart, line chart

Description automatically generated

where TF = EF/kB ~ 104K, as calculated in the free electron file somewhere. It’s interesting that the scattering rate should be so low, thanks to the exclusion principle, since there are way more electrons that a given electron can scatter with, than there are impurities, say. Just for comparison’s sake, what do we expect 1/τ to be? Going back to:



In 3D, EF is given by:



So filling this in,



Problem is a lot of my units are gone, and I have to restore them. I’m missing potentially kB, ℏ, and 4πε0 (left out 4πε0 in the Thermal Equilibrium/Susceptibility file where I got formula for φ(q)), each of which I set to 1 at some point. So actual answer is some,



for some p, q, r. To make the units work out, we need (K is Kelvin):



Seems r must be 0, and p be 2,



Then q = -3 works out. So in SI units, we have:



Alright, filling values, we’ll use kF ~ 1/a ~ 10-9, which is the lattice spacing.



This is roughly the same as the typical impurity scattering rate in metals. Interesting.

**Conductivity via Kubo Formula, more or less**

Just a sketch. I’m doing this in the free space basis for illustration, but won’t we need to use the crystal lattice basis instead? Otherwise, the Umklapp processes would be left out. So we start with our electron – electron Hamiltonian,



and add an electromagnetic field to get, ultimately (as done in electrons + impurity potential folder):



The last term we’ll call VEM. And note definitions:



and,



Our non-equilibrium steady-state distribution function is:



where Ueq includes all terms in H sans VEM. Now we calculate the expectation of the current,



Then filling in VEM we ultimately get:



where,



Taking the Fourier transform, we have that the conductivity tensor is:



and specializing to DC,



and expecting a real σDC, we have:



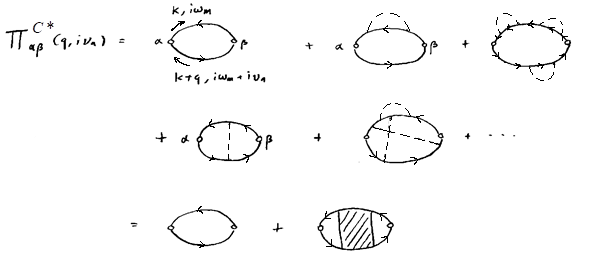
Presuming isotropy, then even more simply,



The current-current correlation function would be given by, as before, without disorder averaging though, because we don’t have any impurities here….



which amounts to, diagrammatically, a structure very similar to what we had before.



Basically just replace the impurity vertex with the e-e vertex. Otherwise, the Feynman rules are the same.