**Meisner Effect**

Now I want to analyze the same situation for superconductors. There are a few different ways to go about it. Note that our results will strictly apply to Type I superconductors, as we’ll be presuming the magnetic field is all either expelled or not. These plots below are for dirty superconductors (i.e., have impurities) I’m thinking?

Chart

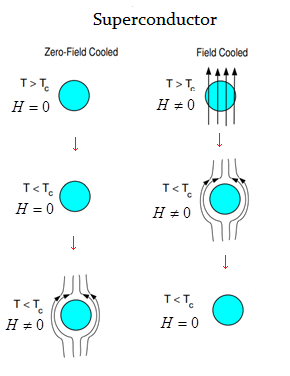
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So should say that Meisner effect has to do with the magnetic susceptibility, plotted above for both Type I’s and Type II’s. From the plot we see that for Type I’s, the susceptibility is χm = -1 up to a critical field. Thus they are perfect diamagnets up to that point. And as we calculate the supercurrents below, keep in mind that one could say that we’re simply calculating the magnetization, as **M** = (1/2)∫d3r **r**×**J** (see EM folder).

Might also point out that even a pure metal has a net paramagnetic response (see Free Day folder), due to the fact that the spin paramagnetic response outweighs the current diamagnetic response. But for small fields at least, we shouldn’t have that here, because the spins enter into singlet Cooper pairs (s = 0).

**Meisner effect via London Theory of Superconductors**

Let’s recall the behavior of an ideal conductor in a changing magnetic field. It was supposed to circulate bound current so as to keep the magnetic flux through it constant. In contrast, a superconductor will circulate bound current so as to keep the magnetic flux zero at all times. This is diagrammed below. Note by H I really mean Bexternal/free, but they’d be the same (well H = μ0Bexternal/free) for suitable geometry (solenoidal, and homogeneous substance, etc). And note the field lines are the total B, not just H.



But it’s said that superconductors expel all magnetic field lines, in both cases. Let’s see how this might be so mathematically. So go back to the current, basically kinematics, equation:



where this time we suppose that just a portion, ns, of the total electron density, n, is superconducting (am presuming we eliminate the last term, ‘cause we’re keeping our analysis restricted to small currents?) and our Maxwell equations:



Again, instead of using the current kinematics equation to solve for jind in terms of E, we’ll do the reverse, and plug into the third equation. Like we did in the previous file, we’ll also neglect the EM wave generating term in the fourth equation, to presume we’re keeping things in equilibrium. And then we have for the third and fourth ME equation:



Working on the top (third ME) equation,



So far this is what metals with zero conductivity do. But to get the Meisner effect, whereby all magnetic flux is expelled, we need to specify the time-independent constant to be zero, so that:



There’s another way to write that current condition.



Well the curl of a gradient is zero, so could say, with a mind to the GL Free Energy stuff,



where for us, φ would just be some function. Now we can combine this with the bottom (4th) Maxwell equation, in the metal’s interior, and say,



which is,



This has solutions of type:



Discounting exponential growth as unphysical, we have exponential decay. And so inside the metal, below the penetration depth,



(This works out to 0.1μm or so), while beyond that, **B**, and the magnetic flux, would be zero.

**Meisner effect BCS theory of Superconductors**

Let’s take a look at the jind equation via BCS theory, and see if we can understand it from a quantum perspective, and try to work out the proportionality constant between **j**ind and **A**, or basically, to get ns as a function of temperature. So we’ll start from Hamiltonian,



where A is the vector potential, and jp the paramagnetic current density,



And from the Metals/Impurities/Nonequilibrium/Conduction/Quantum file, we derived the relationship:



(no ∇φ option here I guess) I’ll call the thing in brackets, **K**(q,ω).



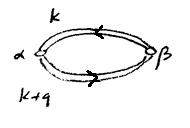
And then we have an equation of the same form as the London equation (well sans the ∇φ part, but …). And so we can equate,



Nice thing about our quantum calculation now is that we can work out the temperature dependence of ns(T). So we’ll start with the current-current correlation function (maybe see the absorbtivity+conductivity tensor file in non-equilibrium properties folder).



Diagrammatically, this is:



where the G’s are the fully interacting (equilibrium, i.e., no A’s) ones. And then the retarded correlation function is,



And now our discussion of the Meisner effect, in the previous file, centered on slowly increasing a uniform magnetic field. So we’ll take the q = ω = 0 limit in this Π equation, and that gives us,



That Trace is:



So,



To evaluate S, we have to do that Matsubara sum over the frequencies. So recall the general technique elaborated on in the Stat Mech Math Appendix,



So now have to evaluate:



There are two poles; moreover they’re double poles. Recall from Math notes/Complex series, that the residue of an nth order pole is (d/dz)n-1[(z-z0)nf(z)]/(n-1)! So



The first guy is:



and second,



So,



At this point might note that:



and so n´F(z) is an even function.



And therefore we can write:



And so all total we have:



Assuming an isotropic medium whereby **K**(0,0) = Kδαβ, we can say,



where ρ(ξ´) is the (*spin-less*) density of states, expressed in terms of ξ = εk - μ´. Now the n´F argument will be very strongly damped about zero since n´F­ exponenetially drops to zero, and so it will fix ξ´ to be approximately zero so we can take ξ´ to be close to zero outside the n´F factor. So we’ll have, where ρF is the density of states at the Fermi surface:



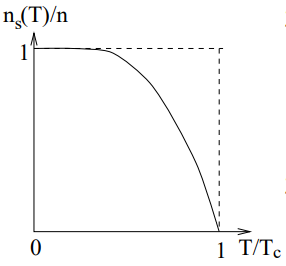
(might find those constants expressions in Free Day/Electrons/Crystal Excitations/Properties) Now let’s compare to the London theory result. We had:



And so we can identify:



Might note that if T = Tc, Δ = 0, and so z = ξ´, and the integral gives 1 → ns(T=Tc) = 0. If T = 0, then n´F → δ(z) or something, but z cannot be 0, and so we just get 0. So ns(T=0) → n.



So there. And from the London Theory, we can now also produce an explicit formula for the penetration depth:

